Design a linear phase 2D low pass FIR digital filter with flat magnitude and sharp transition band with non symmetrical impulse response

Davood Ghaderi, Niloufar Rajabiyoun

1,2 Department of Electrical and Electronic Engineering, Islamic Azad University, International Jolfa Branch, Jolfa, Iran

Abstract: This paper represents a new technique for designing 2D FIR digital filters with flat magnitude and sharp transition band with low arithmetic complexity. Considering wide range usage of filters with these parameters in communication, signal processing, medicine and etc, nowadays optimized designs of these filters has been considered. In addition, filters with these details on cut off band of high pass, band pass and band stop filters with arbitrary pass band should be done extend. In order to flatting the pass band, we used Gaussian approximation in design FIR filters and concluded that ripple in pass band of filters those were designed using Gaussian approximation with attenuation constant coefficients in cut off band, were less than filter designed with Parks_McClellan method and in order to sharpen the transition band, filters frequency response modeled by using of trigonometric functions of frequency and transfer function calculated in time and frequency domains. Typically we cannot design a 2D filter from nonsymmetrical impulse response, then we have create a 2D low pass FIR filter by using of chebyshev polynomials.

Keywords: Two-Dimension FIR filter, Flat Magnitude, Sharp Transition Band, Nonsymmetrical Impulse Response, Chebyshev Polynomials.

1. INTRODUCTION

Different ways has been applied for designing of two-dimensional FIR digital filters. One of them is windows way and another one is frequency transform. McClellan has introduced frequency transform for symmetrical two-dimensional FIR digital filters. By this way one-dimension FIR digital filters transforms to two-dimension filters by symmetrical impulse response by using of transform frequency[1]. Also Karam defined two-dimension filters with complex coefficients by transform frequency. In this way, designed filters could have complex coefficients and also he has generalized those filters to multi-dimension filters. But none of those could not be used in nonsymmetrical filters [2]. In this paper we propose a new method for transforming frequency of one-dimension FIR digital filters with nonsymmetrical impulse response to state of two-dimension FIR digital filters.

2. GAUSSIAN APPROXIMATION METHOD IN DESIGN OF FIR FILTERS

In designing of a digital low pass FIR filter with transition band of \( \omega_c \) and stop edge frequency of \( \omega_s \), filters frequency response present in ideal form in below formula:

\[
H(\omega) = \begin{cases} 
1 & |\omega| \leq \omega_c \\
0 & |\omega| \geq \omega_s 
\end{cases}
\]  

(1)

One of the ways of approximation of ideal frequency response is use of Gaussian functions

\[
H_i(\omega) = \sum C G_{G_i}(\omega) * \delta(\omega - \mu_i)
\]  

(2)

Where \( \delta \) is the impulse unit function, * is the convolution introducer and \( G_{G_i}(\omega) \) is normal (Gaussian) function[3],[4]

\[
G_{G_i}(\omega) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\omega^2 / 2\sigma_i^2}
\]  

(3)

\(\sigma_i\) In above formula is of Gaussian distribution. Collection of Gaussian functions that approximate a FIR filter presented in figure 1.

Parameters \( c, \sigma, \mu \) formula, \( H_i(\omega) \) In should be chosen to minimize the tolerance of approximation. For that, Gappizi and his co-workers suggested [3]-[10]

\[
\sigma = 0.735(\omega_i - \omega_c)
\]  

(4)

In other word, they said \( \sigma \) is a constant coefficient and presented \( \mu_i \) in below formula

\[
\mu_i = i\Delta\mu \quad i = -num, ..., -2, -1, 0, 1, 2, ..., num
\]  

(5)

Where \( \Delta\mu \) was calculated by

\[
\Delta\mu = 2\sqrt{-2\sigma^2 \ln(1-\delta)}
\]  

(6)
In above formula \( \delta \) is the maximum permissible ripple in filters pass band. \( H(\omega) \) in result is
\[
H(\omega) = \frac{1}{2\pi \sigma \text{Max}} e^{-\frac{(\omega - \omega_0)^2}{2\sigma^2}}
\]
(7)

Also
\[
\text{num} = \left[ \frac{\omega}{\Delta \mu} \right] \quad \text{Max} = \frac{\text{num}}{\omega_c} + \frac{1}{2}
\]
(8)

By using of Inverse Fourier Transform, filter impulse response comes from
\[
h(n) = \frac{1}{\pi \text{Max}} e^{-\frac{n^2}{2}} \left[ \sum_{i=1}^{\text{num}} \cos(n \mu_i) + \frac{1}{2} \right]
\]
(9)

Fig. 1 Collection of Gaussian functions that approximate a FIR filter

Fig. 2 presented a digital FIR filter with N=61 (N is FIR filters order), \( \omega_0 = 0.3, \omega_c = 0.33 \) that designed by Gaussian approximation and is showing a comparison with a similar ParksMcClellan method and that showing the ripple in pass band of filters those designed with using of Gaussian approximation to attenuation constant coefficients in cut off band are less than similar filters in which designed with ParksMcClellan method.

3. FILTER DESIGNING

The proposed model for the pseudo-magnitude of the filter transfer function is formulated for equiripple pass band and sharp transition using trigonometric functions of frequency[11],[12]. In the proposed model for a linear phase, equiripple pass band, sharp transition, low pass FIR filter the various regions of the filter response are formulated as follow.

In the pass band region, the frequency response is:
\[
H_{pm}(\omega) = 1 + \text{cos} k_r \omega \quad 0 \leq \omega \leq \omega_p
\]
(10)

Where \( \omega \) the frequency variable, \( H_{pm}(\omega) \) is the pseudo-magnitude of the filter response, \( \delta \) is pass band loss, \( k_r \)

an integer is a filter parameter in the pass band and \( \omega_p \) is the end of ripple channel frequency.

Transition region spans part of the pass band \( (\omega_0 - \omega_p) \) as well as part of the stop band \( (\omega_c - \omega_s) \) where \( \omega_c \) is the cutoff frequency and \( \omega_s \) is the stop band edge frequency.

In the transition region, the frequency response is
\[
H_{pm}(\omega) = \text{Acosk}_s(\omega - \omega_c) \quad \omega_s < \omega < \omega_c
\]
(11)

Where \( k_s \) an integer is a filter parameter in the transition region, \( A \) amplitude parameter and is chosen greater than 1, is the frequency at which \( H_{pm}(\omega) \) equals \( A \). \( \omega_c \) is the frequency at which \( H_{pm}(\omega) \) is zero in the stop band region.

In the stop band region, the frequency response:
\[
H_{pm}(\omega) = -\Delta \sin \frac{k_s}{2} (\omega - \omega_c) \quad \omega_s \leq \omega \leq \pi
\]
(12)

Where \( \Delta \) is the stop band loss, \( k_s \) is the filter parameter in the stop band region.

From (10)
\[
\text{cos} k_r \omega_p = 0
\]
(13)

\[
k_p = \frac{L \pi}{2 \omega_p}
\]
(14)

Where \( L \) is odd, i.e., 1,3,5… to give negative slope due to roll off.

\[
H_{pm}(\omega) = 0 = \text{Acos} \frac{k_r}{2} (\omega_c - \omega_p)
\]
(15)

\[
k_i = \frac{\pi}{2(\omega_c - \omega_p)}
\]
(16)

\[
H_{pm}(\omega) = -\Delta \sin \frac{k_i}{2} (\pi - \omega_c)
\]
(17)

\[
k_c = \frac{\pi}{2(\pi - \omega_c)}
\]
(18)

From (11), \( H_{pm}(\omega) = 1 \) we obtain
\[
A = \frac{1}{\text{cos} \frac{k_i}{2} (\omega_c - \omega_p)}
\]
(19)

Also,
\[
\omega_p = \omega_c - \frac{1}{k_i} \cos^{-1} \left( \frac{1 - \Delta}{A} \right)
\]
(20)

Cut-off frequency
\[
\omega_p = \omega_c + \frac{1}{k_i} \cos^{-1} \left( 1 - \frac{\Delta}{A} \right)
\]
(21)
Stop band edge frequency
\[
\omega_s = \omega_n + \frac{1}{k_r} \cos^{-1}\left(\frac{\delta_1}{2A}\right)
\]  \hspace{2cm} (22)

Transition region width
\[
\omega_s - \omega_n = \frac{1}{k_r} \cos^{-1}\left(\frac{\delta_1}{2A}\cos^{-1}\left(\frac{1-\delta_2}{2A}\right)\right)
\]  \hspace{2cm} (23)

In the stop band,
\[
\omega_s = \omega_n + \frac{4(k_r + 1)\pi}{2k_r}
\]  \hspace{2cm} (24)

Where \(k_r = 0,1,2,\ldots\) choose for \(k_r = 0\) narrowest transition band of the low pass filter. The magnitude response \(H_{pm}(\omega)\) is as shown in figure 3.

3.1 Impulse Response Coefficients

Let \(h(n)\) \(0 \leq n \leq N-1\), be the impulse response of an \(N\)-point linear phase FIR digital filter. The linear phase condition implies that the impulse response satisfies the symmetry condition [7],
\[
h(n) = h(N-1-n), \quad n = 0,1,2,\ldots
\]  \hspace{2cm} (25)

The frequency response for a linear phase FIR filter for odd \(N\) is given by:
\[
H(e^{j\omega}) = e^{-j\frac{N-1}{2}\omega}H_{pm}(\omega)
\]  \hspace{2cm} (26)

Where the pseudo-magnitude response \(H_{pm}(\omega)\) is
\[
H_{pm}(\omega) = h(N-1) + 2 \sum_{i=1}^{\frac{N-1}{2}} h_i N-1 - n \cos n\omega
\]  \hspace{2cm} (27)

The impulse response sequence determined by this frequency response is obtained from

![Impulse Response Coefficients](image)

\[h_j(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{pm}(\omega) d\omega\]  \hspace{2cm} (28)

The impulse response coefficients \(h(n)\) for the resultant filter are obtained by evaluating the integral (28) using equations (10), (11) and (12). Modeling the pseudo-magnitude response \(H_{pm}(\omega)\) in the pass band region, transition region and the stop band region respectively. According that, we obtain the impulse response coefficients as
\[
h(n) = h(N-1-n), \quad n = 0,1,2,\ldots
\]  \hspace{2cm} (29)

\[
\begin{align*}
&\{A, \cos k_1 \omega_1, \sin k_1 \omega_1, \cos k_2 \omega_1, \sin k_2 \omega_1, \ldots, \cos k_r \omega_1, \sin k_r \omega_1, \cos k_{r+1} \omega_1, \sin k_{r+1} \omega_1, \ldots, \\
&2k_1 \sin k_1 \omega_1 &+& \frac{\delta_1}{2k_1} (\cos k_1 \omega_1 - \cos k_{r+1} \omega_1 - \cos k_{r+1} \omega_1)
\end{align*}
\]  \hspace{2cm} (30)

4. Designing Two-dimension FIR Digital Filters With Flat Magnitude and Sharp Transition Band

Transfer function of a one-dimension linear digital FIR filter comes from bellows formula
\[
H(Z) = \sum_{n=0}^{\infty} h(n)Z^{-n}
\]  \hspace{2cm} (31)

Totally \(H(z)\) a filters transformations function suppose with a nonsymmetrical impulse response where as every function could be written as set of one even function and a odd function, therefore \(H(z)\) could be written like bellow
\[
H(Z) = H_e(Z) + H_o(Z)
\]  \hspace{2cm} (32)

Which in it \(H_e(z)\) related to transformation function of a filter with a even symmetrical impulse response and \(H_o(z)\) related to transform function of a filter is a odd symmetrical impulse response. \(H_e(z)\) and \(H_o(z)\) could be reached by bellow form1 2 ual:
\[
H_e(Z) = \frac{1}{2} (H(Z) + Z^{-N} H(Z^{-1}))
\]  \hspace{2cm} (33)

\[
H_o(Z) = \frac{1}{2} (H(Z) - Z^{-N} H(Z^{-1}))
\]  \hspace{2cm} (34)

Filters frequency response comes out from below formula
\[ H(e^{j\omega}) = e^{-j\frac{\pi}{N}N} \left( H_n(\omega) + jH_n(\omega) \right) \]  

(33)

In which in

\[ H_n(\omega) = \sum_{k=0}^{M} a_k \cos(k\omega) \quad (N = 2M) \]

(34)

Now we can develop \( \cos(\omega n) \) according to Chebyshev polynomial

\[ \cos(\omega n) = T_n(\cos(\omega)) \]

(35)

So we can write frequency response of a favorite filter as below

\[ H(e^{j\omega}) = e^{-j\frac{\pi}{N}N} \left( \sum_{k=0}^{M} a_k T_n(\cos(\omega)) + \sqrt{1 - \cos^2(\omega)} \sum_{k=0}^{M-1} b_k T_n(\cos(\omega)) \right) \]

(36)

By above formula clearly you can see we can transform above-mentioned filters to a two-dimension FIR digital filter by replacing \( \cos(\omega) \) with two-dimension function like \( F(\omega_1, \omega_2) \). Every function which comes with \( F(\omega_1, \omega_2) \leq 1 \) condition could be used. But as a some limitation of Karam [2] bellows condition must be applied with above-mentioned condition

\[ F(\omega_1, \omega_2) = -\frac{1}{2} + \frac{1}{2} \cos(\omega_1) + \frac{1}{2} \cos(\omega_2) + \frac{1}{2} \cos(\omega_1) \cos(\omega_2) \]

(38)

5. Simulation Results

In this part we present some of simulation which has been done in this field.

![Figure 5 Two-dimension designed filter in cutoff frequency 0.25\pi](image)

Figure 5 Two-dimension designed filter in cutoff frequency 0.25\pi

![Figure 6 designed filter with conventional ones comparison in cutoff frequency 0.25\pi](image)

Figure 6 designed filter with conventional ones comparison in cutoff frequency 0.25\pi

5. Conclusion

The comparison between Figure 4(a) and 4(b), we can see the magnitude of the filter which designed by frequency transformation method is flatter than pass band of filter which designed by conventional methods. According to [12] with a trigonometric method we can design a sharp transition band filter and according to our method a new model is introduced to produce a 2D low pass FIR filter with flat magnitude and sharp transition band using simple mathematical formulas and linear phase. we concluded that , ripple in pass band of filters designed using Gaussian approximation with attenuation constant coefficients in cut off bands are less than filters designed with Parks-McClellan method. Above mentioned design has been done by MATLAB software, and eventually our results were close to the theoretical results including
ripple maximum 0.005db in pass band and 0.03π in transition band.

REFERENCES


AUTHOR

Davood Ghaderi received the B.S and M.S. degrees in Electrical Engineering from Islamic Azad University, Tabriz Branch, Iran, in 2006 and 2009, respectively. From 3 years ago he had begun a wide research in field of ideal filters in FIR & IIR domains and expanding them to optical devices. His favorite fields are DSP, filters and image and signal processing and have several papers and survey projects in these

Niloufar Rajabiyoun received the B.S. and M.S. degrees in Electrical Engineering from Engineering Faculty of Islamic Azad University, Tabriz Branch, Iran, in 2006 and 2009, respectively. Her main field in M.S. was speech signals and Blind Source Separation (BSS) analysis and has extensive study in these fields with more than 10 different papers and survey projects. She had taught in different branches of Islamic Azad University and private institutes of technology in Iran, from four years ago.