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Exploring Effects of Rewiring Strategies Based on Network Topological Properties

Abdulla F. Ally[1] ,Zhang Ning[2]

[1, 2] University of Shanghai for Science and Technology, Business School
Shanghai 200093, China.

Abstract

While small networks can be visualized directly by its graph, larger networks can be envisaged and described by using a set of statistical measures of a network's topology. In this paper, we devote our attention on three quantitative measures namely average path length, clustering coefficient, and degree distribution to investigate and discuss how rewiring strategy affects degree of spreading in random networks. In particular, we used Fermi function to account for stochastic dynamics, whereby a random network is rewired to generate a new network that serve as a basis for comparisons with the original. We argue that when network connections are rewired, the topological properties may change considerably due to the rewiring probability of some nodes' connections. While random networks portray a Poisson distribution, the new network, by contrast, shows an increasing divergence from binomial distribution. The distribution is positively skewed and looks like exponential distribution. Most nodes have very small degree, and a few have large degree. As such, the probability of nodes being informed is higher due to larger clustering of nodes and shorter paths between them. We notice that the degree of spreading is further increased by rewiring strategies with positive β. This implies that the degree of spreading in random networks depends on the rewiring strategies—the value of the function's parameter –used.

Keywords: network dynamics, rewiring,spreading dynamics

1. Introduction

Understanding the spreading process in networks has been an important factor in modelling networks and spreading dynamics [1]. In network paradigm, information spreads from one individual to another through social ties amongst the population. In the case of networks with a wide-ranging degree distribution, it is argued that most high connected individuals are good candidates in enhancing information spreading [2], [3]. However, there are rare circumstances under which high degree nodes have low impact on the spreading process [4]. In network science, selection of individual to spread information in networks is usually modelled using link rewiring strategies [5]–[7]. Several studies have, traditionally, designed this stochastic process in terms of probability functions with their parameters used as the strength of selecting neighbours to connect based on the payoff difference [8], [9].

During edge rewiring, the topology of networks may change significantly. While small networks can be visualized directly by its graph, larger networks can be envisaged and described by using a set of quantitative performance measures [10]. In this paper, we focus on three robust measures of a network's topology namely average path length, clustering coefficient, and degree distribution to investigate how rewiring strategies affect degree of spreading in random networks. In particular, we designed a rewiring model based on Fermi function, whereby random network is rewired to generate a new network that serve as a basis for comparisons with the original network. By quantifying the aforementioned quantities in each network topology, we can estimate the effect of the strategy used to rewire the network.

2. Methods and Experimental Settings

The simulation was performed on random network with a total population N = 4000 and average degree k = 4. To enable the spreading in the network, we integrated the rewiring model with Susceptible-Infected-Recovered (SIR) model [11]-[13]. This model has been widely used in the literature to describe disease spreading as well as rumours and information spreading in social networks [14]. In the SIR model, the individual can be in one of the three states [11]: Susceptible state (S-node) being state of individuals who are liable to be informed, Informed state (I-node) as the state of those who have information and can communicate to S nodes, and Recovered state (R-node) described as a state of individuals who have information but exhausted and thus will not take part in the spreading process.

The study assumed that the total population, N, is constant, and that at time $T = 0$, the beginning of the computer simulation, one individual is randomly selected as a seed (I node) and the other (N-1) nodes are all susceptible (S nodes). In the discrete formulation of the SIR model [15], [16], we assume at each time step informed individuals communicate information to their susceptible neighbours at a rate of 0.3 and recover at a fixed time. Each informed individual breaks the existing link with its neighbours if the neighbour's payoff is less than a randomly selected individual in the population. Accordingly, information spreads in the network until it

reaches the steady state when there are only recovered and susceptible individuals.
In this paper, the rewiring between nodes is designed in such a way that informed node breaks existing link with its neighbours and rewires the link to a randomly selected susceptible node from the whole network, with rewiring probability determined by Fermi function *F*[9], [17], [18].

$$F = \frac{1}{1 + \exp^{-\beta(\Pi_s - \Pi_{s'})}} \quad (1)$$

The parameter β represents strength of selecting *S* neighbours, with values taken from (-∞, ∞). Larger values signify stronger selection, which indicate selection of high degree nodes and verse visa. The notations $\Pi_{S'}$ and Π_S represent payoffs of the two targeted nodes: a node *S'* which is linked to an informed node and a node *S* to be rewired, respectively. The term payoff has been widely used in different studies to represent total number of respective entities. For example, Perc et al [19] considered the term payoffas the number of players adopting one's strategy in the spatial Prisoner's dilemma game. In dynamic systems, at each time step every player interacts with its neighbours, and acquires a certain number of players, which they call it as payoff. They designed that each player change its strategy by comparing the number of players it has (its payoff) and the number of players of its neighbours (neighbour's payoff) following the strategies adoption function of the payoff difference. In dynamic social networks, payoff is described as the total number of friends an individual has in the population.

3. Simulation Results and Discussion

To illustrate the information spreading dynamics, we simulated different topologies of random network by quantifying degree distribution, average path length, and clustering coefficient, which are the most important quantities for characterising network structure's connectedness and cliquishness [20]. The approach we used was to construct another network by rewiring random structure using the Fermi function (Eq. 1) integrated with the SIR model and compare the new network with the initial random structure based on the aforementioned quantities. This enabled us to study how rewiring strategies affect degree of spreading. Traditionally, quantitative descriptions of network topology have been postulated as a particularly appealing framework for understanding characteristics of the network [10]. To motivate this discussion, we calculated the clustering coefficient and average path length of the initial random network (before rewiring). In this work, the initial random structure exhibits clustering coefficient of 0.02 and an average path length of 4.268. We argue that when network connections are rewired, clustering coefficients and average path length may change considerably due to the rewiring probability of some nodes' connections. For the simulation with controlling parameters (β = -3, 0 and 3) of Fermi function, we show that the new network structure has shorter average path length and larger clustering coefficients for all the rewiring strategies of β (Table 1) as compared to the initial random structure.
It has been said that a shorter average path length facilitate information spreading as everyone is connected to everyone else through a short path [10]. However, larger clustering coefficient also signifies high degree of spreading, as the probability of individualsbeing informed is high as well.

Table 1: Average clustering coefficients (*C*) and average path lengths (*l*) of the rewired structure simulated with different rewiring strategies.

Rewiring Strategies	*C*	*l*
β = -3	0.620	2.085
β = 0	0.959	2.031
β = 3	0.960	2.033

Clustering describes how many of node's contacts also have contact among each other. High clustering of individuals means more local spread (within cliques) and thus a rapid local depletion of susceptible nodes. In extreme cases, spreading gets trapped within highly cohesive clusters. Intuitively, initial structure random network should be less conducive for the spread as compared to the rewired network.
In order to gain further credence, we explored similar analysis as above but this time it is based on degree distribution in order to investigate patterns of networks connectivity. In traditional random networks (before rewiring) most nodes have a medium node degree. The degrees of all nodes are distributed around the average (7.987) as shown in Figure 1.

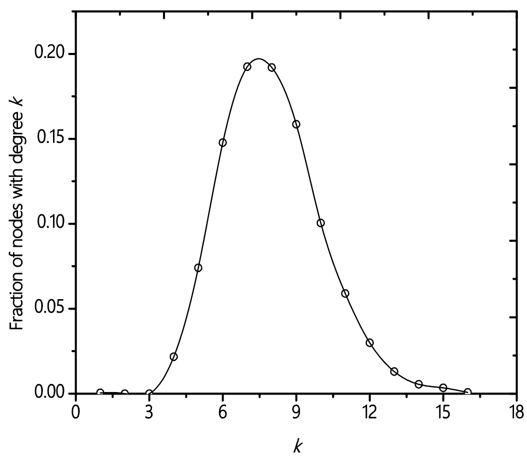

Figure 1: Degree distribution of random network with degree *k*, before rewiring its edges.

It can be seen that nodes in the rewired network structure, by contrast, show an increasing divergence from binomial distribution (Figure 2). The distribution is positively skewed and looks like exponential distribution. Most nodes have very small degree, and a few have large degree. As such, the probability of nodes being informed grows. We notice that the degree of spread is further increased by rewiring strategies with positive β. This implies that the degree of spreading in networks depends on the rewiring strategies—the value of the function's parameter— used.

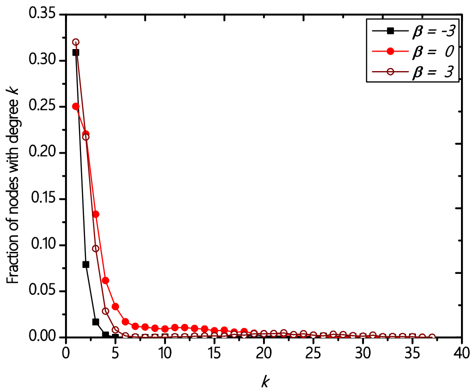

Figure 2: Degree distribution of the new network after rewiring random network with Fermi function.

4.Concluding Remarks

It has been revealed that a few network properties exist by which the network topology can be determined. Most important topological properties are average path length, clustering coefficient and degree distribution. In this work, we compared the aforementioned quantities from different network topologies in order to examine how rewiring strategies affectdegree of spreading in networks. Specifically, by employing Fermi function, random network was rewired to generate a new network that serves as the basis for comparisons with the initial structure. It is argued that when network connections are rewired, the topological properties may change considerably due to the rewiring probability of some nodes' connections.

The degree distribution of the new network structure is radically different from Poisson distribution observed in many random networks. The distribution is positively skewed and looks like exponential distribution. Most nodes have very small degree, and a few have large degree. As such, the probability of nodes being informed is higher due to larger clustering of nodes and shorter paths between them. We notice that the degree of spread is further increased by rewiring strategies with positive β. In view of this, rewiring strategy is shown to be relevant factor influencing the degree of spreading. This work more likely influences the success of spreading strategies.

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References

[1]. K. Maksim, K. G. Laros, H. Shlomo, L. Fredrik, M. Lev, H. E. Stanley, H. A. Makse, "Identification of influential spreaders in complex networks," Nature Physics, 6, pp. 888–893, 2010.
[2]. K. Tuyls,S. Parsons, "What Evolutionary Game Theory tells us about Multi-agent Learning," Artificial Intelligence, 171, pp. 406-416, 2007.
[3]. Y. Wu-Jie,X. Cheng-Yi, "Role of Investiment Heterogeneity in the Cooperation on Spatial Public Goods Game," PLoS ONE, 9, pp. e91012, 2014.
[4]. F. Fu, D. I. Rosenbloom, L. Wang, M. A. Nowak, "Immitation Dynamics of Vaccination Behavior on Social Networks," Proceedings of the Royal Society B: Biological Sciences, 278, pp. 42-49, 2011.
[5]. T. Gross, C. J. D. D'Lima, B. Blasius, "Epidemic dynamics on an adaptive network," Physical Review Letter, 96, pp. 208701, 2006.
[6]. M. G. Zimmermann,V. M. Eguíluz, "Cooperation, social networks, and the emergence of leadership in a prisoner's dilemma with adaptive local interactions," Physical Review E, 72(5), pp. 056118, 2005.
[7]. L. B. Shaw,I. B. Schwartz, "Fluctuating epidemics on adaptive networks," Physical Review E, 77, pp. 066101, 2008.
[8]. A. Traulsen,C. Hauert, "Stochastic Evolutionary Game Dynamics," in Reviews of Nonlinear Dynamics and Complexity, S. HG, (eds.), Germany: Wiley-VCH Verlag GmbH & Co. KGaA, 2009.
[9]. G. Szabó,C. Tőke, "Evolutionary Prisoner's Dilemma Game on a Square Lattice," Physical Review E, 58, pp. 69–73, 1998.
[10]. R. Albert,A.-L. Barabási, "Statistical Mechanics of Complex Networks," Reviews of Modern Physics, 74(1), pp. 47-97, 2002.
[11]. R. M. Anderson,R. M. May, Infectious disease of humans: Dynamic and Control, Oxford Press,UK, 2002.
[12]. H. W. Hethcote, "The Mathematics of Infectious Diseases," SIAM Review, 42, pp. 599-653, 2000.
[13]. E. Volz,L. A. Meyers, "Epidemic Thresholds in Dynamic Contact Networks," Journal of the Royal Society Interface, 6, pp. 233-241, 2009.
[14]. C. Castellano, S. Fortunato, V. Loretto, "Statistical Physics of Social Dynamics," Reviews of Modern Physics, 81, pp. 591-646, 2009.

[15]. L. F. Lago-Fernández, R. Huerta, F. Corbacho, J. A. Sigüenza, "Fast response and temporal coherent oscillations in small-world networks," Physical Review Letter, 84(12), pp. 2758-2761, 2000.
[16]. R. Parshani, S. Carmi, S. Havlin, "Epidemic threshold for the susceptible-infectious-susceptible model on random networks," Physical Review Letter, 104, pp. 258701, 2010.
[17]. X.-J. Chen, F. Fu, L. Wang, "Interaction Stochasticity Supports Cooperation in Spartial Prisoner's Dilemma," Physical Review E, 78, pp. 051120, 2008.
[18]. M. Perc,A. Szolnoki, "Co-evolutionary Games-A Mini Reviews," Biosystems Engineering, 99, pp. 109-125, 2010.
[19]. M. Perc,M. Marhl, "Evolutionary and dynamical coherence resonances in the pair approximated prisoner's dilemma game," New Journal of Physics, 8(8), pp. 142, 2006.
[20]. M. E. J. Newman, "The Structure and Function of Complex Networks," SIAM Review, 45, pp. 167-256, 2003a.