MORPHOLOGICAL FILTERS USING SIMULATED ANNEALING TECHNIQUE

P K Kumaresan

Professor Computer Science and Engineering VMVK Engineering College (Vinayaka Missions University)
Salem 636308 Tamil Nadu

Abstract

Nonlinear filters are becoming an increasingly popular area of image and signal processing. They are often better than linear filters in removing noise without distorting image features. Among these nonlinear filters, morphological filters are one of the most important and useful, providing an important class of image and signal processing operators. Another well-known class of non-linear filters is the class of rank order filters. Soft morphological filters are a combination of morphological and weighted rank order filters. Soft morphological filters improve the behavior of standard morphological filters in noisy conditions. But design methods existing for these filters tend to be computationally intractable or require some special knowledge. Heuristic based approach- Simulated annealing provide a useful tool for the design and optimization of soft morphological filters. In this paper, a new method for the soft morphological filter design using simulated annealing is proposed and illustrated by empirical results. It is found that the performance of the filter is better under MSE error criterion than MAE criterion.

Keywords: Soft Morphology, Soft Dilation, Soft Erosion, Filter Optimization, Simulated Annealing

1. INTRODUCTION

Mathematical morphology is a powerful tool for solving image-processing problems, such as image enhancement, noise removal, feature extraction, texture analysis etc. Morphological operations have been viewed as filters for which the properties have been studied[1]. Another well-studied class of nonlinear filters are rank order filters. Soft morphological filters are a combination of morphological and weighted rank order filters [2], which have been introduced to improve the behavior of traditional morphological filters in noisy environments. The idea is to slightly relax the typical morphological definitions in such a way that a degree of robustness is achieved, while most of the desirable properties of typical morphological operations are maintained. Soft morphological filters are less sensitive to additive noise and to small variations in object shape.

Soft morphological filters have three parameters, which define their behavior, and they all contribute to the statistical and detail preservation properties of the filter. Unfortunately, these are no analytical criteria for choosing these parameters. Hence, heuristic based methods for finding optimal or near optimal Soft morphological filters with respect to various error criteria (MAE and MSE) are presented in this paper.

2. SOFT MORPHOLOGICAL OPERATIONS

Soft morphological operations are nonlinear signal transforms, which are related to discrete flat morphological operations stemming from mathematical morphology [3]. The soft morphological operations (filters) adopt the order statistics to replace the maximum and minimum in standard morphological operations. The basic idea of soft morphological operations is that the structuring set *B* is split into two subsets: *hard center A* (i.e., *A* ∨ *B)* and the soft boundary *B\A***,** where '\', denotes the set difference.

The **Soft Dilation** of the Structuring system [B, A, r] is defined by

Soft Dil ($X_1, X_2, \ldots X_N$;[B A r])= the r^{th} largest value of the multiset $\{r \Diamond X_i \ : \ i \vee A\} \cup \{X_i : i \vee (B\backslash A)\}$

The **Soft Erosion** of the Structuring system [B,A,r] is defined by

Soft Ero ($X_1, X_2, \ldots X_N$;[B A r])= the r^{th} smallest value of the multiset $\{r \Diamond X_i \ : \ i \vee A\} \cup \{X_i : i \vee (B\backslash A)\}$

Morphological **opening** is a compound operator consisting of erosion followed by dilation; similarly, morphological **closing** is dilation followed by erosion**.**

3. OPTIMIZATION BY SIMULATED ANNEALING ALGORITHM

3.1. SIMULATED ANNEALING
3.2.

Simulated annealing (SA) is a generic probabilistic meta-algorithm for the global optimization problem, namely locating a good approximation to the global optimum of a given function in a large search space[4]. The name and inspiration come from annealing in metallurgy, a technique involving heating and controlled cooling of a material to increase the size of its crystals and reduce their defects. The heat causes the atoms to become unstuck from their initial positions (a local minimum of the internal energy) and wander randomly through states of higher energy; the slow cooling gives them more chances of finding configurations with lower internal energy than the initial one. In the simulated annealing (SA) method, each point *s* of the search space is compared to a *state* of some physical system, and the function *E*(*s*) to be minimized is interpreted as the internal *energy* of the system in that state. Therefore the goal is to bring the system, from an arbitrary *initial state*, to a state with the minimum possible energy. At each step, the SA heuristic considers some *neighbors* of the current state *s*, and probabilistically decides between moving the system to state *s'* or staying put in state *s*. The probabilities are chosen so that the system ultimately tends to move to states of lower energy. Typically this step is repeated until the system reaches a state which is good enough for the application, or until a given computation budget has been exhausted. The neighbors of each state are specified by the user, usually in an application-specific way

3.2. Applying SA for Soft morphological filter

The search *space* or *solution set* of **soft** morphological filters consist of structuring sets and repetition parameter r. Thus, the optimization problem is a combinatorial optimization problem in which we have to find optimal combination of structuring set elements, ***(i.e.***, hard center and soft boundary) and repetition parameter ***r.*** More formally, we have a pair ***(S ,*** c), where S is the search space and c: ***S—›R*** is a *cost function*, which is defined by the optimization criterion. The problem is to find an item $\mathbf{s_o}$∨S such that c($\mathbf{s_o}$) <= c(s), for every s∨S. In this problem, the cost function is *mean absolute error* (MAE) or *mean square **error*** (MSE), and every item **s** in the search space is of the form S ***= (W , r)*** where $W=(w_1, w_2,..w_n)$, r∨Z+ , w_i∨{0,1,2}, *i* ∨*1,2,...,n.*

The elements w_i are the structuring set elements, and their values are interpreted ***as*** follows:

- w_i= 0 means that the structuring element in the ith place does not belong to the structuring system, ***i.e.*** it has no effect to the result of the filtering.
- w_i= 1 means that the structuring element in the ith place is part of the soft boundary of the structuring set.
- w_i= 2 means that the structuring element in the ith place is part of the hard center of the structuring set.
- Metropolis algorithm is used for applying SA to soft morphological filter. In this algorithm a sequence of configurations (here structuring systems) is generated until the global optimum is reached. Each new configuration is selected from the neighborhood of the previous configuration.
  Perturbation Scheme: (neighborhood of the previous configuration)
  New [B A r] will be formed by following one of the following options:
  Add or subtract 1 to/from the value of r randomly select one element in the structuring system ,

- If that element is '0' change it to '1' or '2'.
- If that element is '1' change it to '0' or '2'.
- If that element is '2' change it to '0' or '1'.

The selected configuration is then accepted with some non-zero probability which depends on the value of the control parameter and on the difference of the values of the costs of the previous and the selected configurations. Here either MAE or MSE criteria is used as the cost function.

Simulated annealing algorithm is described as follows:

begin
initialize control parameter *t*, let $t = t_0$
select starting configurations, let s = s_0
calculate error(cost) function *E(s)*
while(t > end value) do
begin
for(i = 0; I < k; i++)
Create a new configuration s' at random
if E(s') <= E(s) , then s = s' ,
else set s = s' with probability $e^{-(E(s')-E(s))/t}$

t = t * a
end
end

When the algorithm terminates, s is the desired configuration and E(s) is the corresponding optimum value. The initial value for the control parameter is usually chosen in a way such that in the beginning almost all configurations would be accepted in the algorithm. The value of a lies in the range of (0, 1) and is usually set near 1 for getting good results.

4. EMPIRICAL RESULTS OF SIMULATED ANNEALING BASED SOFT FILTERS

Simulated annealing technique is implemented for the soft filter design. The test results obtained with two different images are provided.

4.1 CAMERAMAN IMAGE

In this section, some empirical results found by applying simulated annealing technique are presented. When the image is corrupted by impulsive noise, search for optimal structuring systems for soft dilation, soft erosion, soft closing or soft opening is done. The MSE and MAE are used as the optimization criteria. The initial structuring system selected is 5 x 5 square. Figure 2 is the original image of "camera man". Figure 3 is the image impregnated with impulsive noise. The filtering result of soft opening optimized by applying simulated annealing under MAE and MSE criteria are shown in Figure 4 and Figure 5 respectively.

On analyzing the results based on PSNR value, the filter performances are better under MSE error criterion than MAE criterion. The results obtained by applying SA can be improved by setting the control parameter to a high value.

Fig. 2. Original image

Fig. 3. Image impregnated with impulsive noise

Fig. 4. Result of optimal soft opening by applying SA (under MAE criteria PSNR 15.07)

Fig.5. Result of optimal soft opening by applying SA (under MSE criteria PSNR 18.21)

4.2 LENA IMAGE

An extensive study of soft filter performance was taken up with Lena image. The parameters, initial temperature (t_0), number of repeats (k) and end value for 't' , in the algorithm are found by trial and error. The tunable parameters are initialized in a random configuration and the annealing algorithm is run using a cooling schedule whereby the temperature is reduced by a factor of a=0.95 until it reached the end value (t_0 is set as 50, then no: of iterations: -31).The number of repeats (k) is typically around 20 per iteration so that a total of 620 annealing steps are performed.

The search of optimal structuring systems for soft opening is done in the cases, where the image is corrupted with 3, 6 and 10 percent of impulse noise. Several experiments for each of these cases is made where the optimal structuring set included in the 3x3 and 5x5 square.

Fig. 8 shows the original and noisy image of Lena and Fig. 9 shows the filtering result of soft opening under MAE and MSE criterion with N=3x3.

MAE_o - mean absolute error between original and noisy image

MAE_f - mean absolute error between original and filtered image

MSE_o - mean square error between original and noisy image

MSE_f - mean square error between original and filtered image

Fig 8 (a) Original Image of 'Lena'

Fig 8 (b) Image impregnated with 3% impulse noise

Fig 9 (a) Filtering result of soft opening under MAE criterion with N=3x3

Fig 9 (b) Filtering result of soft opening under MSE criterion with N=3x3.

Five independent tests are performed and the final value of the comparison criteria for each noise ratio are obtained as average of these runs and the results obtained are summarized in table 4.2.1 and table 4.2.2.

As SA method is probabilistic in nature, it must be run several times to obtain reliable results. The results in table 4.2.3 revealed that the filter performance is improved when control parameter value is set to a high value.

Table 4.2.1 Optimal structuring system for Soft opening w.r.t. MAE

Noise ratio	Structuring system	Lena	
		MSE_o	MSE_f
3	B=[1 0 0;1 2 2;1 0 0]; r=4;	545.8628	39.3647
6	B=[0 1 2;1 2 2;2 1 1]; r=5;	1.1233e+003	86.7689
10	B=[2 0 0;1 2 0;1 0 1]; r=4;	1.8552e+003	144.7921

Table 4.2.2 Optimal structuring system for Soft opening w.r.t. MSE

Noise ratio	Structuring system	Lena	
		MAE_o	MAE_f
3	B=[1 1 1;1 2 0;0 2 1]; r=3;	4.7576	3.2189
6	B=[1 2 2;1 2 1;2 0 1]; r=5;	7.7135	4.1134
10	B=[0 2 0;1 2 0;1 0 1]; r=3;	12.7456	10.4123

Table 4.2.3 The optimal parameters with respect to MAE and MSE for different control parameter values in 'Lena' image

Control parameter ' t '	MAE		MSE	
	cost	PSNR	cost	PSNR
20	4.3202	26.6133	74.3539	26.8913
35	2.7394	27.8465	66.6819	27.3806
50	3.2189	28.4893	39.3647	28.6696

5. CONCLUSION

In this paper, heuristic based techniques have been developed for the design and optimization of soft morphological filters. The method optimizes filters with respect to criterion based on mean absolute error (MAE) and mean squared error (MSE) using simulated annealing method. The technique applied is capable of finding grey-scale soft morphological filters which have good performance in the removal of impulsive noise, whilst retaining the necessary fine details within the original image. It is found that the performance of the filter is better under MSE error criterion than MAE criterion. The results presented herein can also be used to improve the understanding of the effects of the parameters of soft morphological filters. The structuring system [B A r] has a pronounced effect on the performances of soft morphological filters, and its optimization results in better filtering.

REFERENCES

[1]. R.L. Stevenson and G.R. Arce, "Morphological filters: Statistics and further syntactic properties," IEEE Trans. Circuits Syst., Vol.34 PP.1292-1305, Nov. 1987
[2]. L.KosKinen, J.Astola and Y.Neuvo, "Soft morphological filters," Journal of Mathematical Imaging and Vision,No.5,pp.231-262,May 2002
[3]. L. Koskinen, J. Astola, and Y. Neuvo, "Soft morphological filters,"in Proceedings of SPIE Symp. on Image Algebra and Morphological Image Processing II, San Diego, USA, pp. 262-270,July 1992.
[4]. S.Kirkpatrick, C.D.Gelatt,Jr., M.P.Vecchi, "Optimization by Simulated Annealing", Science, vol.220, No.4598, pp.671-680, May 1983.
[5]. R.M.Haralick, "Image analysis using mathematical morphology," IEEE Trans. On Pattern Analysis and Machine Intelligence, vol. 9, no. 4, pp. 532-550, July 1987.
[6]. Rafael C. Gonzalez, Richard E.Woods. 'Digital Image Processing', Pearson Education, Second edition, 2002.