

Split and Non Split Two Domination Numbers of Semi Total - Point graph.

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Abstract: A dominating set $D \subset V$ of a graph G is a Split two dominating set if the induced subgraph $\langle V-D \rangle$ is disconnected and every vertex is adjacent to atleast two vertices in D . The Split two domination number $\gamma_{s2}(G)$ is the minimum cardinality of a Split two dominating set. A Dominating set $D \subset V$ of a graph G is a Non Split two dominating set if the induced subgraph $\langle V-D \rangle$ is connected and every vertex in $V-D$ is adjacent to atleast two vertices in D . The Non Split two domination number $\gamma_{ns2}(G)$ is the minimum cardinality of a Non Split two dominating set. For any graph $G = (V, E)$, the semi total point graph $T_2(G) = H$ is the graph whose vertex set is the union of vertices and edges in which two vertices are adjacent if and only if they are adjacent vertices of G or one is a vertex and other is an edge of G incident with it. In this paper we investigate the behavior of Split two and Non Split two domination number for Semi Total - point graphs.

Keywords: Split Two Domination Number, Non Split Two Domination Number, & Semi Total - Point Graph.

1. INTRODUCTION

Euler introduced the concept of graph theory, in the year 1736. He created the first graph to simulate a real time place and situation to solve a problem which is known as Seven Bridges of Konigsberg. Historically, the first domination-type problems came from chess. In the 1850s, several chess players were interested in the minimum number of queens such that every square on the chess board either contains a queen or is attacked by a queen (recall that a queen can move any number of squares horizontally, vertically, or diagonally on the chess board). In 1862, the chess master C.F. de Jaenisch wrote a treatise [9] on the applications of mathematical analysis to chess in which he considered the number of queens necessary to attack every square on a $n \times n$ chess board. The study of dominating sets in graphs was started by Ore and Berge [1, 5]. The domination number and the independence number were introduced by Cockayne and Hedetniemi [4]. In 1997 and 2000, the concept of the split and non split two domination was introduced by Janakiram and Kulli [2,3]. Though it is very young, it has been growing fast and has numerous applications in various fields. And Dr. A. Mydeen Bibi., introduced the concept of Split two and Non Split two

domination of a graph [6]. The concept of Semi Total - point graph was introduced in [7].

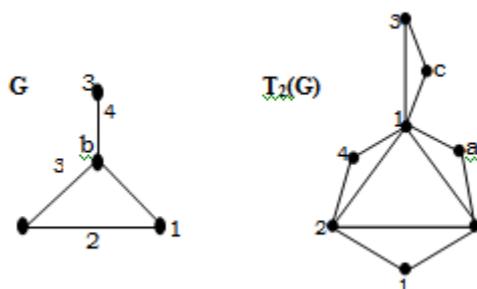
Domination in graphs has been an extensively researched branch of graph theory. Let $G = (V, E)$ be a simple undirected graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by $d(u)$. A Subset D of V is called a dominating set in G if every vertex in $V-D$ is adjacent to at least one vertex in S . The minimum cardinality taken over all dominating sets in G is called the domination number of G and is denoted by $\gamma(G)$. A Dominating set is said to be two dominating set if every vertex in $V-D$ is adjacent to atleast two vertices in D . The minimum cardinality taken over all, the minimal two dominating set is called two domination number and is denoted by $\gamma_2(G)$. A Dominating set $D \subset V$ of a graph G is a Split (Non Split) two dominating set if the induced subgraph $\langle V-D \rangle$ is disconnected (connected) and every vertex in $V-D$ is adjacent to atleast two vertices in D . The Split (Non Split) two domination number $\gamma_{s2}(G)$ [$\gamma_{ns2}(G)$] is the minimum cardinality of a Split [Non Split] two dominating set.

1.1 Preliminaries

In this section, we provide a brief summary of the definitions and other results which are prerequisites for the present work. All the graphs considered here are simple finite, undirected without loops and multiple edges. Let $G = (V, E)$ be a graph and as usual we denote $n = |V|$ and $m = |E|$.

Definition 1.2

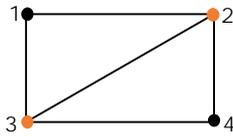
For any graph $G=(V,E)$, the semi total point graph $T_2(G) = H$ is the graph whose vertex set is the union of vertices and edges in which two vertices are adjacent if and only if they are adjacent vertices of G or one is a vertex and other is an edge of G incident with it.



Definition 1.4

A Dominating set $D \subset V$ of a graph G is a **Split two dominating set** if the induced subgraph $\langle V-D \rangle$ is disconnected and every vertex in $V-D$ is adjacent to atleast two vertices in D . The Split two domination number $\gamma_{s2}(G)$ is the minimum cardinality of a Split two dominating set.

Example 1.5

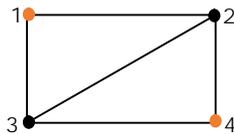


$D=\{2, 3\}$ and $V-D=\{1, 4\}$ hence $V-D$ is disconnected, $\gamma_{s2}(G)=2$

Definition 1.6

A Dominating set $D \subset V$ of a graph G is a **Non Split two dominating set** if the induced subgraph $\langle V-D \rangle$ is connected and every vertex in $V-D$ is adjacent to atleast two vertices in D . The Non Split two domination number $\gamma_{ns2}(G)$ is the minimum cardinality of a Non Split two dominating set.

Example 1.7



$D=\{1, 4\}$ and $V-D=\{2, 3\}$ hence $V-D$ is connected $\gamma_{ns2}(G)=2$

1.8 Basic Definitions

- A **Path P_n** is a walk in which all the vertices are distinct.
- A simple graph with 'n' vertices ($n \geq 3$) and 'n' edges is called a cycle graph if all its edges form a cycle of length 'n'.
- If the **degree of each vertex in the graph is two**, then it is called a **Cycle Graph**.
- Any cycle with a pendant edge attached at each vertex is called **crowns graph** and is denoted by C_n^+ .
- A simple graph in which there exists an edge between every pair of vertices is called a **complete graph**.
- A **wheel graph W_n** of order n, sometimes simply called an n-wheel, n is a graph that contains a cycle of order $n - 1$ and for which every vertex in the cycle is connected to one other vertex.
- A **complete bipartite graph** is a simple bipartite graph such that two vertices are adjacent if and only if they are in different partite sets. The complete bipartite graph with partite sets of size m and n is denoted $K_{m,n}$.
- A **triangular snake** graph is obtained from a path v_1, v_2, \dots, v_n by joining v_i and v_{i+1} to a new vertex w_i for $i = 1, 2, \dots, n-1$ and denoted by mC_3 -snake.
- Any path with a pendant edge attached at each vertex is called **Hoffman tree** and is denoted by P_n^+ .
- The **Book B_n** is the graph $S_n \times P_m$ where S_n is the star with $n+1$ vertices.

▪ The **friendship graph** or **Dutch t-windmill**, denoted by F_n can be n constructed by identifying n copies of the cycle C_n at a common vertex.

▪ A **Fan graph F_n** is defined as the graph $K_m + P_n$ join , where K_m is the empty graph on m nodes and P_n is the path graph on n nodes Any fan with a pendant edge attached at each vertex is denoted by F_n^+ .

▪ The **Friendship graph**, denoted by C_m^n can be constructed by identifying n copies of the cycle C_a at a common vertex.

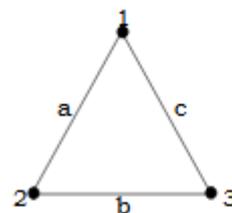
▪ Take $P_2, P_4, \dots, P_{n-2}, P_n, P_n, P_{n-2}, \dots, P_4, P_2$ paths on $2, 4, \dots, n-2, n, n, n-2, \dots, 4, 2$ vertices and arrange them centrally horizontal, where $n \equiv 0(mod 2), n \neq 2$. A graph obtained by vertical vertices of given successive paths is known as **Plus graph** of size n, denoted by Pl_n .

2.Split And Non Split Two Domination Of Semi Total Point Graph

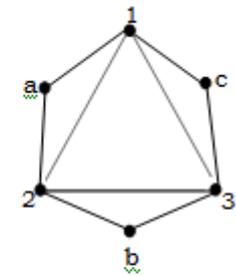
➤ For all cycles, $\gamma_{s2}(T_2(C_n)) = n$ for $n \geq 3$ and

$$\gamma_{ns2}(T_2(C_n)) = n \text{ for } n \geq 3$$

C_3



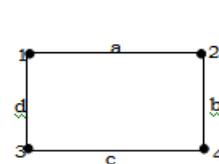
$T_2(C_3)$



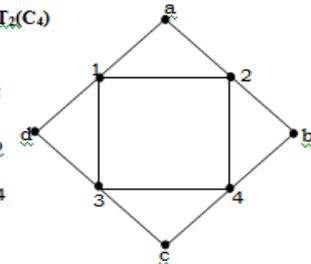
$$\gamma_{s2}(T_2(C_3)) = 3$$

$$\gamma_{ns2}(T_2(C_3)) = 3$$

C_4



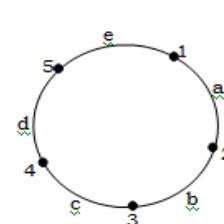
$T_2(C_4)$



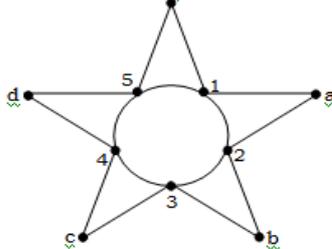
$$\gamma_{s2}(T_2(C_4)) = 4$$

$$\gamma_{ns2}(T_2(C_4)) = 4$$

C_5



$T_2(C_5)$



$$\gamma_{s2}(T_2(C_5)) = 5$$

$$\gamma_{ns2}(T_2(C_5)) = 5$$

- or all paths, $\gamma_{s2}(T_2(P_n)) = n$ for $n \geq 4$ and
 $\gamma_{ns2}(T_2(P_n)) = n+1$ for $n \geq 4$
- For all complete graphs, $\gamma_{s2}(T_2(K_n)) = n$ for $n \geq 3$ and

$$\gamma_{ns2}(T_2(K_n)) = \frac{n(n-1)}{2} \text{ for } n \geq 3$$
- For all wheel graphs, $\gamma_{s2}(T_2(P_n)) = n+1$ for $n \geq 3$ and

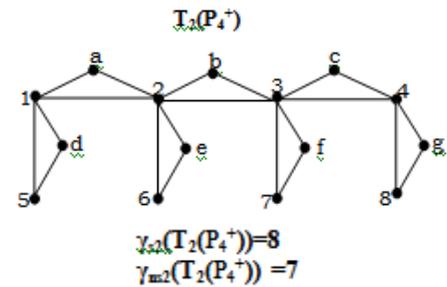
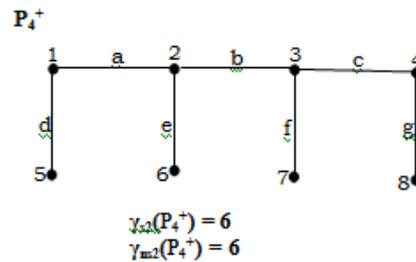
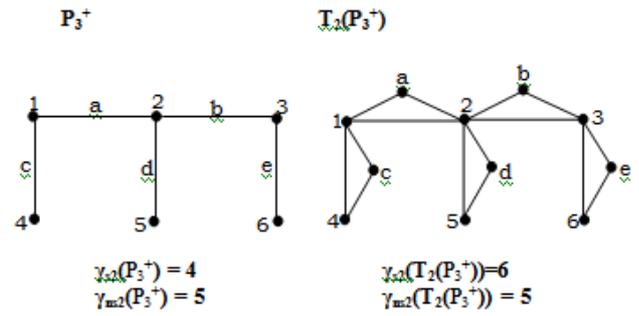
$$\gamma_{ns2}(T_2(P_n)) = 2(n-1) \text{ for } n \geq 3$$
- For all complete bipartite graphs,
 $\gamma_{s2}(T_2(K_{m,n})) = m+n$ for $m, n \geq 2$ and
 $\gamma_{ns2}(T_2(K_{m,n})) = mn$ for $m, n \geq 2$
- For all book graphs, $\gamma_{s2}(T_2(B_n)) = 2(n+1)$ for $n \geq 2$ and

$$\gamma_{ns2}(T_2(B_n)) = 3n+1 \text{ for } n \geq 2$$
- For all friendship graphs,
 $\gamma_{s2}(T_2(C_m^n)) = 2n+1$ for $n \geq 2, m=3$ and
 $= 4n$ for $n \geq 2, m=4$
 $\gamma_{ns2}(T_2(C_m^n)) = 3n$ for $n \geq 3, m=3$ and
 $= 3n+1$ for $n \geq 2, m=4$
- For all plus graph $\gamma_{s2}(T_2(PI_n)) = \frac{n^2}{2} + n$ for
 $n \geq 4$ and
 $\gamma_{ns2}(T_2(PI_n)) = n^2$ for $n \geq 4$
- For all book graphs, $\gamma_{s2}(T_2(B_n)) = 2(n+1)$ for
 $n \geq 2$ and

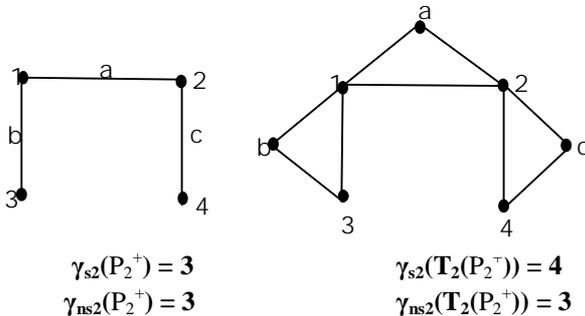
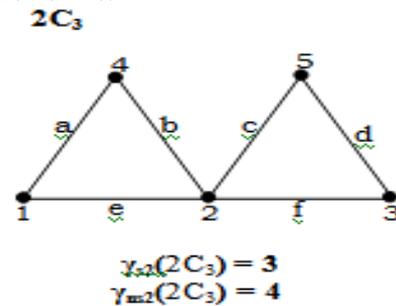
$$\gamma_{ns2}(T_2(B_n)) = 3n+1 \text{ for } n \geq 2$$
- For all crown graphs, $\gamma_{s2}(T_2(C_n^+)) = 2n$ for $n \geq 3$
and

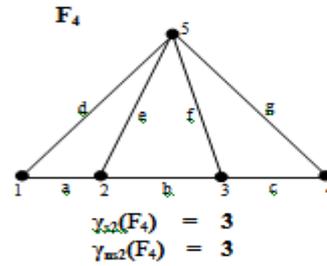
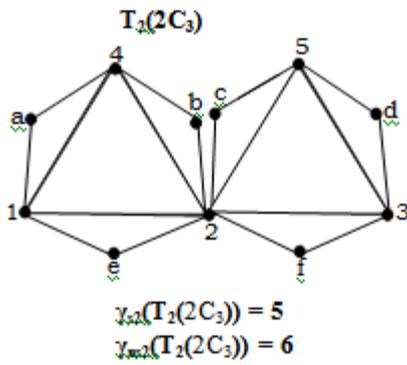
$$\gamma_{ns2}(T_2(C_n^+)) = 2n \text{ for } n \geq 3$$
- For all Hoffman trees, $\gamma_{s2}(T_2(P_n^+)) = 2n$ for $n \geq 2$
and

$$\gamma_{ns2}(T_2(P_n^+)) = 2n-1 \text{ for } n \geq 2$$

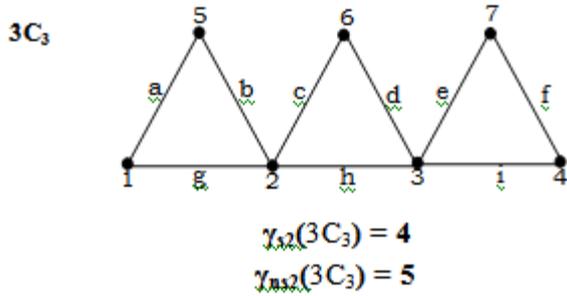
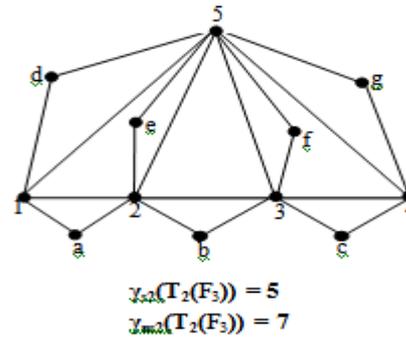


- For all triangular snake, $\gamma_{s2}(T_2(mC_3)) = 2m+1$ for $n \geq 2$
 $\gamma_{ns2}(T_2(mC_3)) = 3m$ for $n \geq 2$

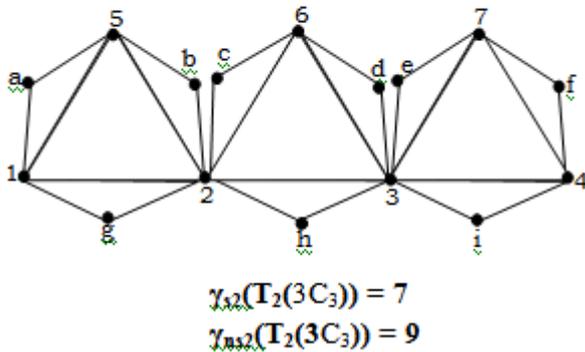




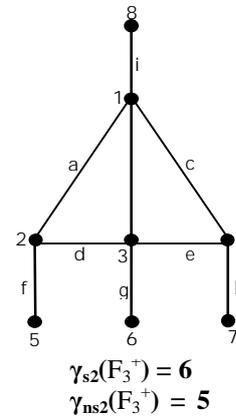
$T_2(F_4)$



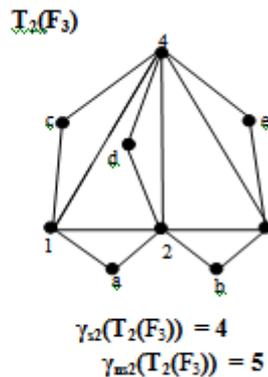
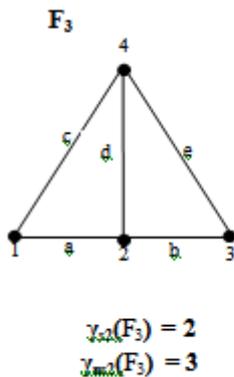
$T_2(3C_3)$



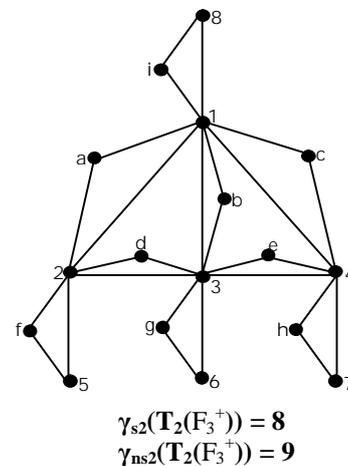
➤ For all fan graphs $\gamma_{s2}(T_2(F_n^+)) = 2n+2$ for $n \geq 3$ and $\gamma_{ns2}(T_2(F_n^+)) = 3n$ for $n \geq 3$

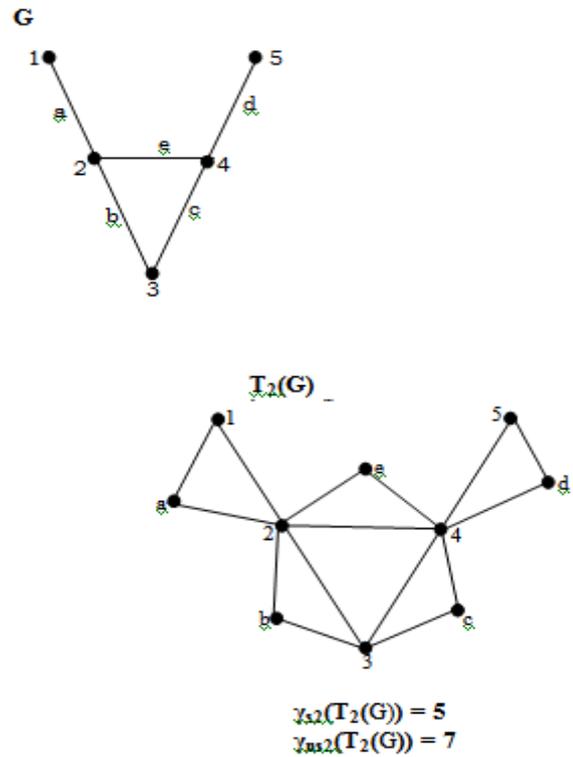
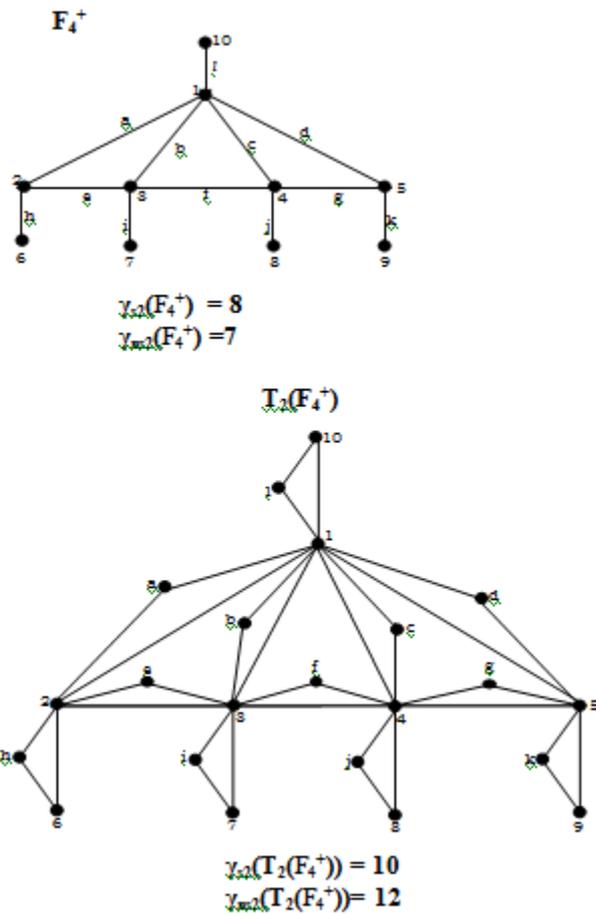


➤ For all fan graphs $\gamma_{s2}(T_2(F_n)) = n+1$ for $n \geq 3$ and $\gamma_{ns2}(T_2(F_n)) = 2n-1$ for $n \geq 3$



$T_2(F_3^+)$



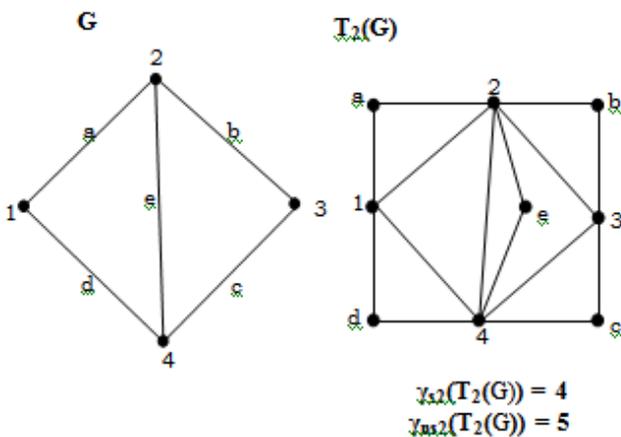


3.3 Moser- Spindle:

The Moser-graph (also called Moser's spindle) is an undirected graph, with 7 vertices and 11 edges. It is a unit distance graph requiring 4 colors in any graph coloring.

3. Split And Non Split Two Domination Of Some Special Semi Total Point Graph

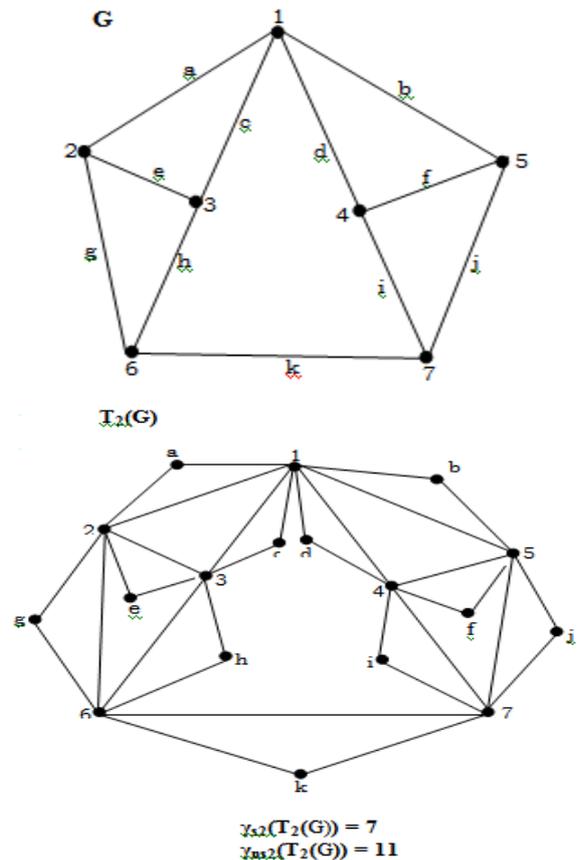
3.1 Diamond Graph



3.2 Bull- Graph:

The bull graph is a planar undirected graph with 5 vertices and 5 edges in the form of a triangle with 2 disjoint pendant edges.

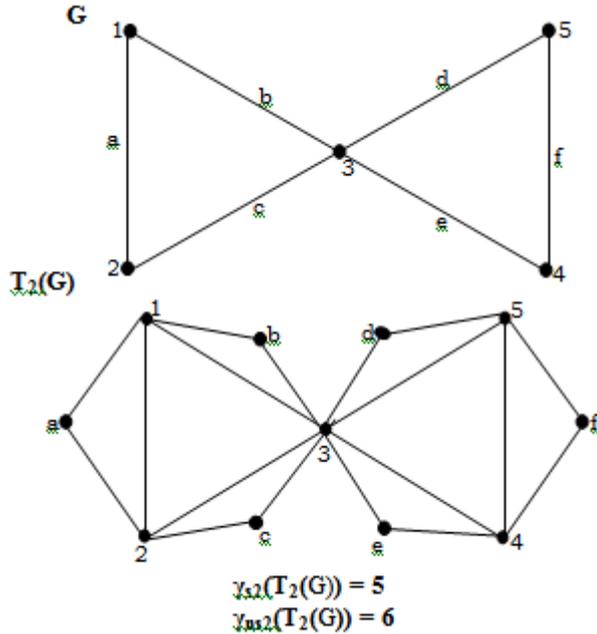
It has chromatic number 3, chromatic 3, radius 3, diameter and girth 3. It is also a block graph, a split graph, an interval graph, a claw-graph, a 1-vertex connected graph and a 1- edge- connected graph.



3.4 Butterfly- Graph:

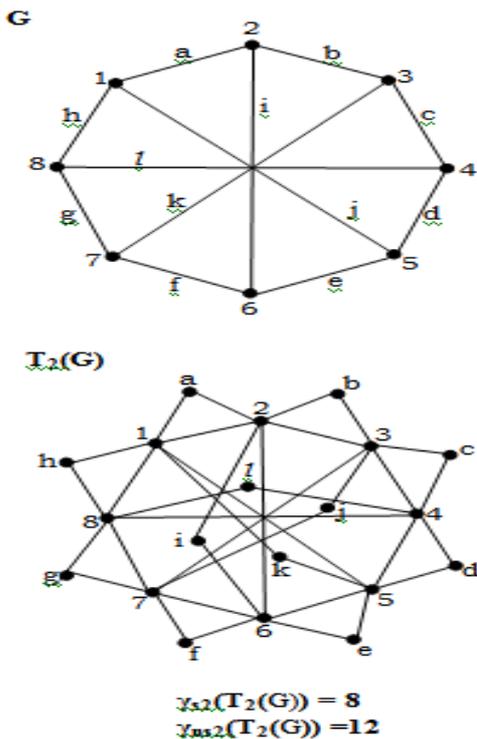
A graph is bowtie-free if it has no butterfly as an induced subgraph. The triangle-free-graph are bowtie-free graph, since every butterfly contains a triangle.

The full automorphism group of the butterfly graph is a group of order 8 isomorphic to the Dihedral group D_4 , the group of symmetries of a square including both rotation and reflection.

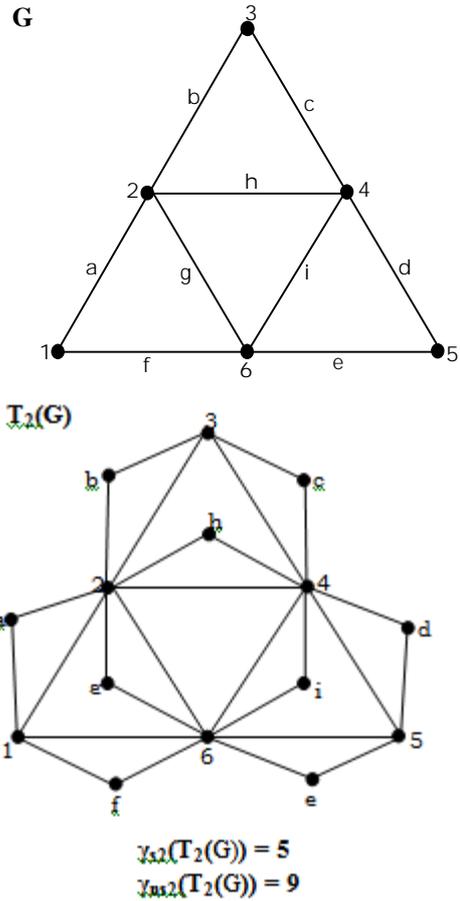


3.5 Wagner Graph:

The Wagner graph is a 3-regular graph with 8 vertices and 12 edges. It is the 8 vertex Mobius ladder graph.

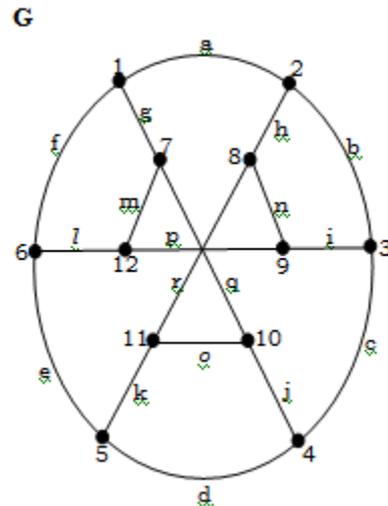


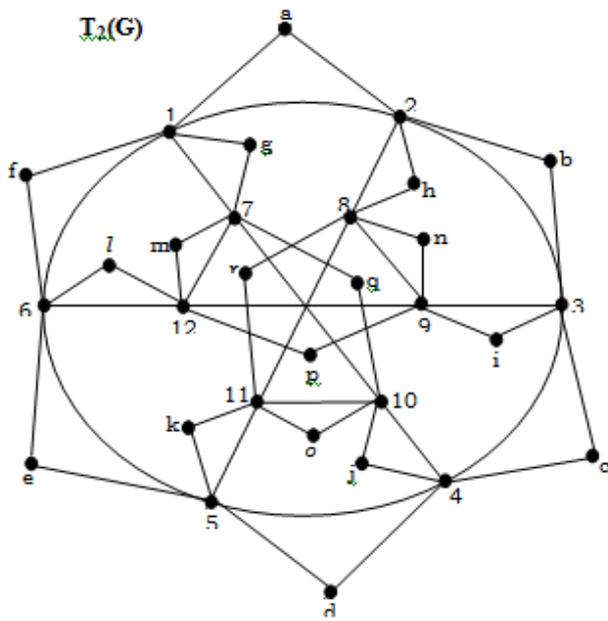
3.6 Hajos - Graph :



3.7 Franklin- Graph:

The franklin graph is a 3-regular graph with 12 vertices and 18 edges. It is hamiltonion and has chromatic number 2, chromatic index 3, radius 3, diameter 3, and girth 4. It is also a 3-vertex connected and 3 edge- connected perfect graph.



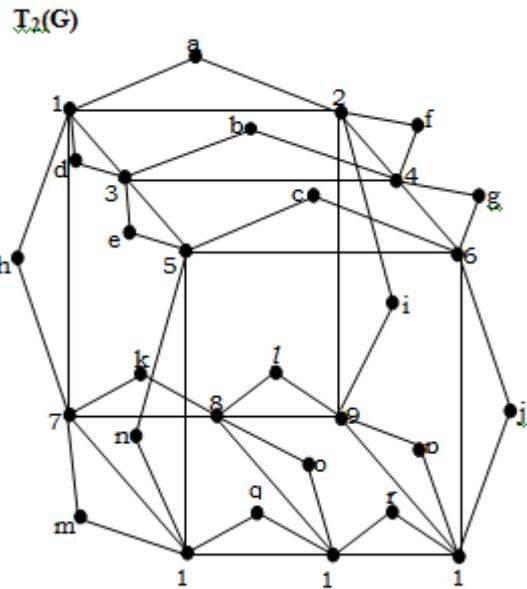
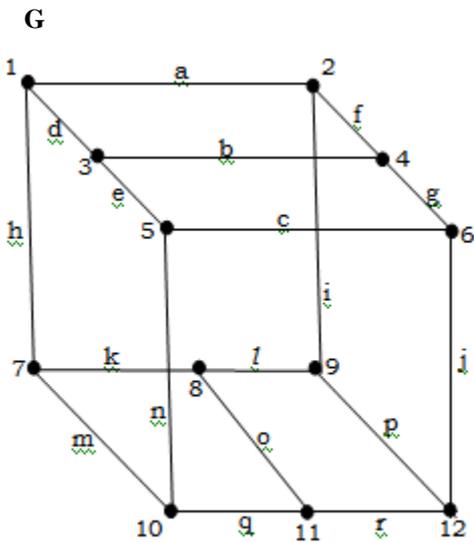


$$\chi_{12}(T_2(G)) = 12$$

$$\chi_{18}(T_2(G)) = 18$$

3.8 Bidiakis - Cube :

The Bidiakis-cube is a cubic Hamiltonian graph and can be defined by the LCF notation $[-6,4,-4]^4$. The bidiakis cube can also be constructed the top and bottom faces which connect the centres of opposite sides of the faces. The two additional edges need to be perpendicular to each other.

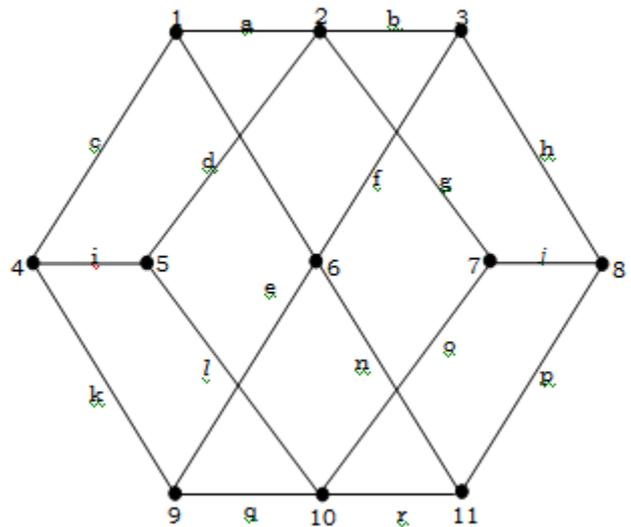


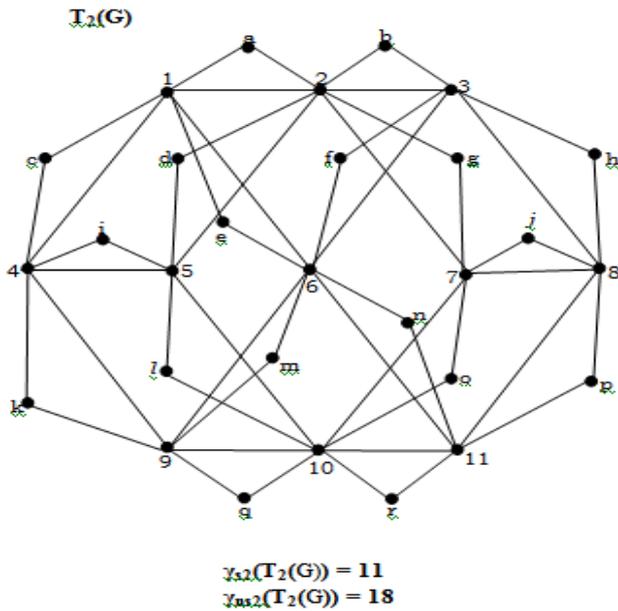
3.9 Herschel- Graph:

The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges, the smallest non-hamiltonian polyhedral graph.

The Herschel graph is a planar graph it can be drawn in the plane with none of its edges crossing. It is also 3-vertex connected. The removal of any 2 of its vertices leaves a connected subgraph.

G

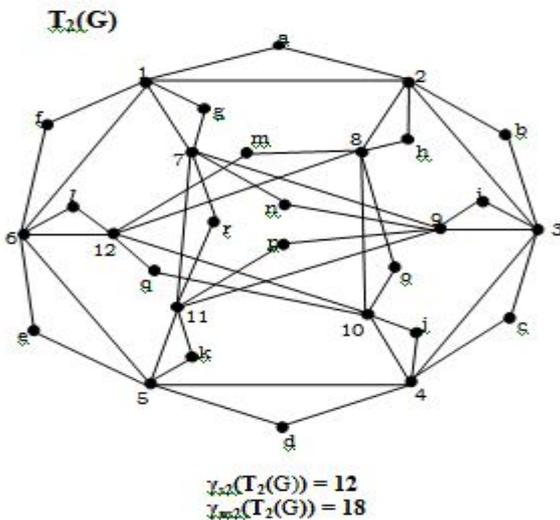
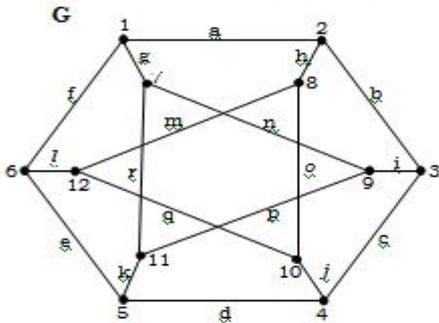




3.10 Durer- Graph:

Durer's solid is combinatorially equivalent to a cube with 2 opposite vertices truncated although durer's depiction of it is not in this form but rather as a truncated rhombohedron or triangular truncated trapezohedron.

The durer graph is the graph termed by the vertices & edges of the durer solid. It is a cubic graph of girth 3 & diameter 4.



4. Main Results

Observation 4.1:

$\gamma_{s2}(T_2(G)) \leq \gamma_{ns2}(T_2(G))$. C_n is the graph for which $\gamma_{s2}(T_2(G)) = \gamma_{ns2}(T_2(G)) = n$.

Observation 4.2:

For any semi total point graph $T_2(G)$, $\gamma(T_2(G)) \leq \gamma_{s2}(G)$, and $\gamma(T_2(G)) \leq \gamma_{ns2}(T_2(G))$

Proof: Since every Split two dominating set is a dominating set of $T_2(G)$, $\gamma(T_2(G)) \leq \gamma_{s2}(T_2(G))$, similarly, every Non Split two dominating set is a dominating set of $T_2(G)$, $\gamma(T_2(G)) \leq \gamma_{ns2}(T_2(G))$.

Theorem 4.3: For any path graph P_n ; $\gamma_{s2}(T_2(P_n)) = n$, and $\gamma_{ns2}(T_2(P_n)) = n+1$ for $n \geq 4$

Proof: Let $\{v_1, v_2, \dots, v_n\}$ be a vertex set of P_n and $\{e_1, e_2, \dots, e_{n-1}\}$ be a edge set of P_n & $V(T_2(P_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_{n-1}\}$ be a vertices of $T_2(P_n)$

Let $D = \{v_1, v_2, \dots, v_n\}$ be a minimal dominating set of $T_2(P_n)$. Then $V-D = \{e_1, e_2, \dots, e_{n-1}\}$. Every vertex in $V-D$ is adjacent with exactly two vertices in D and also $V-D$ is disconnected. Hence $\gamma_{s2}(T_2(P_n)) = n$, for any $n \geq 4$.

Let $D = \{e_1, e_2, \dots, e_{n-1}\}$ be a minimal dominating set of $T_2(P_n)$. Then $V-D = \{v_1, v_2, \dots, v_n\}$. Every vertex in $V-D$ is adjacent with exactly two vertices in D and also $V-D$ is connected. $\gamma_{ns2}(T_2(P_n)) = n+1$ for any $n \geq 4$.

Theorem 4.4: For all cycles, $\gamma_{s2}(T_2(C_n)) = n$, and $\gamma_{ns2}(T_2(C_n)) = n$ for $n \geq 3$.

Proof : Let $V(C_n) = \{v_1, v_2, \dots, v_n\}$, $E(C_n) = \{e_1, e_2, \dots, e_n\}$, be the vertex and edge set of C_n & $V(T_2(C_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$ be a vertices of $T_2(C_n)$.

Let us take $D = \{v_1, v_2, \dots, v_n\}$. Then $V-D = \{e_1, e_2, \dots, e_n\}$. Clearly, every vertex in $V-D$ is adjacent with exactly two vertices in D and also $V-D$ is disconnected. Hence $\gamma_{s2}(T_2(C_n)) = n$ for any $n \geq 3$. Now $D = \{e_1, e_2, \dots, e_n\}$ Then $V-D = \{v_1, v_2, \dots, v_n\}$. Clearly, every vertex in $V-D$ is adjacent with exactly two vertices in D and also $V-D$ is connected. Hence $\gamma_{ns2}(T_2(C_n)) = n$ for any $n \geq 3$.

Theorem 4.5: For all complete graphs, $\gamma_{s2}(T_2(K_n)) = n$, and $\gamma_{ns2}(T_2(K_n)) = \frac{n(n-1)}{2}$ for $n \geq 3$

Proof : Let $V(K_n) = \{v_1, v_2, \dots, v_n\}$ & $E(K_n) = \{e_1, e_2, \dots, e_n\}$ $V(T_2(K_n)) = \{v_1, v_2, \dots, v_n, e_1, e_2, \dots, e_n\}$ be a vertices of $T_2(K_n)$.

Let us take $D = \{v_1, v_2, \dots, v_n\}$ Then $V-D = \{e_1, e_2, \dots, e_n\}$. Clearly, every vertex in $V-D$ is adjacent with exactly two vertices in D and also $V-D$ is disconnected and $|D| = n$.

Hence $\gamma_{s2}(T_2(C_n)) = n$ for any $n \geq 3$. Now $D = \{e_1, e_2, \dots, e_n\}$ Then $V-D = \{v_1, v_2, \dots, v_n\}$. Clearly, every vertex in $V-D$ is adjacent with exactly two vertices in D and also $V-D$ is connected. Since there are $\frac{n(n-1)}{2}$ edges in the complete graph K_n we get, $|D| = \frac{n(n-1)}{2}$. Hence $\gamma_{ns2}(T_2(C_n)) = n$ for any $n \geq 3$.

Theorem 4.6: For any semi total point graph G , $\gamma(T_2(G)) \leq \min \{\gamma_{s2}(T_2(G)), \gamma_{ns2}(T_2(G))\}$

Proof : Since every Split two dominating set and every Non Split two dominating set of $T_2(G)$ are the dominating

set of $T_2(G)$, we have $\gamma(T_2(G)) \leq \gamma_{s2}(T_2(G))$ and $\gamma(T_2(G)) \leq \gamma_{ns2}(T_2(G))$ and hence $\gamma(T_2(G)) \leq \min\{\gamma_{s2}(T_2(G)), \gamma_{ns2}(T_2(G))\}$.

Theorem 4.7: For any semi total point graph, Split two dominating set is minimal if and only if for each vertices $u, v \in D$, one of the following condition is satisfied:

- There exists a vertex $w \in V-D$ such that $N_{s2}(w) \cap D = \{u, v\}$
- v is not an isolated vertex in $\langle D \rangle$
- $\langle V-D \cup \{u, v\} \rangle$ is connected.

Proof: Suppose D is a minimal split two dominating set such that $\{u, v\}$ does not satisfy any of the above conditions. Then by (a) and (b) $D - \{u, v\}$ is a domination set, also since (c) is not satisfied, $\langle V-D \rangle$ is disconnected. Therefore $D - \{u, v\}$ is a Split two dominating set contradicting the minimal of D . Hence v satisfies one of the above conditions, and the bound is sharp.

5. CONCLUSION

The concept of split two domination and non split two domination in graphs relates dominating sets and the disconnectivity and connectivity of $V-D$ respectively. The split two and non split two domination numbers of some standard graphs are already available in the literature while we investigate the split two and non split two domination number for the semi total point graph. We can also extend the result for all semi total point graphs, and semi total block graphs too.

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