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Kushare Transform of Error Functions in Evaluating Improper Integrals.

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Abstract: ***In this paper we study Kushare transform of error functions and use it to solve the inproper integrals.***

Keywords: Kushare transform, Error functions, Improper integrals, Integral transforms.

1. INTRODUCTION

Recently, Integral transforms are one of the mostly used simple mathematical technique to obtain the solutions of advance problems of space, science, technology, engineering , commerce and economics . The important feature of these integral transform is to provide exact solution of of problem without lengthy calculations.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, in September 2021, Kushare and Patil [1] introduced Kushare transform for facilitating the process of solving differential equations in time domain. Further in October 2021 Khakale and Patil [2] introduced Soham transform. As researchers are introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform to solve the problems on Newton's law of Cooling .

In April 2022 D.P. Patil , et al [4] used Kushare transform for solving the problems on growth and decay . In October 2021, D.P. Patil [5] used Sawi transform in Bessel functions. Further, Patil [6] used Sawi transform of error functions to evaluate improper integrals. Laplace transforms and Shehu transforms are used to Patil [7] in chemical sciences. Patil [8] solved wave equation by using Sawi transform and its convolution theorem. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].

Solution of wave equation is obtained by using double Laplace and double Sumudu transforms by D .P. Patil [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Laplace, Elzaki and Mahgoub transforms are compared and used for solving system of first order and first degree by Kushare and Patil [12] . D. P. Patil [13] used Aboodh and Mahgoub transform for solving boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu , Aboodh , Elazki and Mahagoub transform and used it for solving boundary value problems by Patil et al [15]. Patil et al [16] used Emad-Sara transform for solving Volterra Integral equations of first kind. Futher Patil with Tile and Shinde [17] used transform for solving Volterra integral equations for first kind. Rathi sisters and D. P. Patil [18] used Soham transform for solving system of differential equations. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation.

Kandalkar, Zankar and Patil [20] used general integral transform of error function for evaluating improper integrals. Dinkar Patil, Prerana Thakare and Prajakta Patil used double general integral transform for the solution of parabolic boundary value problems [21]. Patil used Emad-Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform in Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Patil and Wagh[27] used Soham transform in chemical sciences. Derle, Rahane and Patil [28] used double rangaig transform. Raundal and Patil[29] used double general integral transform for solving the boundary value problems. Patel, Khelukar and Patil [30] used Alenzi transform for solving growth and decay problems. Patil and Malunjkar [31] introduced double Kushare transform. D. P. Patil et al [32] used Emad Sara transform in population growth and decay problems.

This paper is organized as follows: Second section is reserved for preliminary concepts required to solve improper integrals. We state and prove some important properties of Kushare transform in third section. Fourth section is reserved for statement and proof of Convolution theorem for Kushare transform. Properties of error function are states in same section. Fifth section contains Kushare transform of error function. Applications are in sixth section and lastly conclusion is drawn.

2. PRELIMINARIES:

In this section we state some preliminary concepts.

2.1 Definition Kushare Transform:

An integral transform said to be KUSHARE transform which changes the characterized for capacity of outstanding request to think about capacities in the set A characterized by

A={f(t)/ ∃M, T_1, T_2 >0,|f(t)<Me $^{\frac{|t|}{Tj}}$,if t∈$(1)^j$× [0,∞)}……(1)

and is defined as,

K[f(t)]=s(v)=v$\int\int_0^\infty f(t)e^{-tv^\propto}$dt,>t≥0,$T_1$≤v≤$T_2$….(2)

For a given function in set A, the constant M must be finite number, T_1, T_2 may be finite or infinite .

2.2 Kushare Transform of the Some Preliminary Functions:

1. $K(1) = \frac{1}{v^{\alpha-1}} = 1 = S(v)$

Inversion formula $K^{-1}\left(\frac{1}{v^{\propto-1}}\right) = 1 = f(t)$

2.$K(t^n) = \frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}} = S(v)$

Inversion formula:$K^{-1}\left(\frac{\Gamma(n+1)}{v^{\alpha(n+1)-1}}\right) = t^n = f(t)$

3.$K(e^{at}) = \frac{v}{v^{\alpha}-a} = S(v)$

Inversion Formula: $K^{-1}\left(\frac{v}{v^{\alpha}-a}\right) = e^{at} = f(t)$

4.$K(\sin at) = \frac{av}{(v^{2\alpha}+a^2)} = S(v)$

Inversion Formula: $K^{-1}\left(\frac{v}{(v^{2\alpha}+a^2)}\right) = \frac{\sin at}{a} = f(t)$

5.$K(\cos(at)) = \frac{v^{\alpha+1}}{(v^{2\alpha}+a^2)} = S(v)$

Inversion Formula: $K^{-1}\left(\frac{v^{\alpha+1}}{(v^{2\alpha}+a^2)}\right) = \cos(at) = f(t)$

Kushare Transform of derivatives:

If K[f(t)] = S(v) then,

1.$K[f'(t)] = v^{\alpha}S(v) - vf(0)$

2.$K[f''(t)] = v^{2\alpha}S(v) - v^{\alpha+1}f(0) - v\,f'(0)$

3. $K[f^n(t)] = v^{n\alpha}\,S(v) - v\sum_{k=0}^{n-1} v^{\alpha(n-k-1)}\,f^k(0)$

3. PROPERTIES OF KUSHARE TRANSFORM:

In this section we state and prove some properties of Kushare transform.

3.1. Change of scale property:

If Kushare transform of function $F(t)$is $S(v)$then Kushare function $F(at)$is given by

$$K[f(at)] = a^{\alpha^{-1}-1}s\left(\frac{v}{\sqrt[\alpha]{a}}\right).$$

Proof: we know by definition of Kushare Transform

$$K[f(t)] = v\int_0^\infty f(t)e^{-tv^\alpha}$$

Now,

$$K[f(at)] = v\int_0^\infty f(at)e^{-tv^\alpha}$$

Put,$a\,t = p \implies t = \frac{a}{p}$

$$a\,dt = dp$$

$$K[f(at)] = v\int_0^\infty \frac{1}{a}f(p)e^{\frac{-p}{a}v^\alpha}dp$$

$$= \frac{v}{a}\int_0^\infty f(p)\,e^{-P\left(\frac{v}{\sqrt[\alpha]{a}}\right)^\alpha}dp$$

$$= \frac{v}{a}\frac{\sqrt[\alpha]{a}}{\sqrt[\alpha]{a}}\int_0^\infty f(p)e^{-P\left(\frac{v}{\sqrt[\alpha]{a}}\right)^\alpha}dp$$

$$= \frac{a^{1/\alpha}}{a}\frac{v}{\sqrt[\alpha]{a}}\int_0^\infty f(p)\,e^{-P\left(\frac{v}{\sqrt[\alpha]{a}}\right)^\alpha}dp$$

$$K[f(at)] = a^{\alpha^{-1}-1}s\left(\frac{v}{\sqrt[\alpha]{a}}\right)$$

Hence proved.

3.2 Shifting Property:

If, Kushare transform of function $F(t)$ is $S(v)$ then Kushare transform of $e^{at}f(t)$ is given by $K[e^{at}F(t)] = \frac{v}{(v^{\alpha}-a)^{1/\alpha}}\,S\left(\sqrt[\alpha]{(v^\alpha - a)}\right)$

Proof: We know by Kushare transformation definition,

$$K[f(t)] = v\int_0^\infty f(t)e^{-tv^\alpha}dt$$

Now,

$$K[e^{at}F(t)] = v\int_0^\infty e^{at}\,F(t)e^{-tv^\alpha}dt$$

$$= v\int_0^\infty F(t)\;e^{at-tv^\alpha}dt$$

$$= v\int_0^\infty F(t)\,e^{-t\langle v^\alpha-a\rangle}dt$$

$$= v\int_0^\infty F(t)\,e^{-t\langle\sqrt[\alpha]{v^\alpha-a}\rangle^\alpha}dt$$

$$= \frac{v}{\sqrt[\alpha]{v^\alpha-a}}\sqrt[\alpha]{v^\alpha-a}\int_0^\infty F(t)e^{-t\left(\sqrt[\alpha]{v^\alpha-a}\right)^\alpha}dt$$

$$K[e^{at}F(t)] = \frac{v}{(v^\alpha-a)^{1/\alpha}}\,S\left(\sqrt[\alpha]{(v^\alpha-a)}\right)$$

3.3 Kushare Transform of function $tf(t)$:

If$K[F(t)] = S(v)$, then $K[tF(t)] = \left(\frac{1}{\alpha v^{\alpha-1}}\right)\left[\frac{d}{dv} - \frac{1}{v}\right]S(v)$.

Proof: Let,$K[F(t)] = S(v)$

By the definition of Kushare Transformation, we have

$$K[f(t)] = v\int_0^\infty f(t)e^{-tv^\alpha}$$

Now, differentiate this equation with respect to v,we get

$$\frac{d}{dv}S(v) = \frac{d}{dv}\left(v\int_0^\infty F(t)e^{-tv^\alpha}dt\right)$$

$$= \int_0^\infty F(t)\frac{d}{dv}ve^{-tv^\alpha}dt$$

$$= \int_0^\infty F(t)\left[ve^{-tv^\alpha}(-t\alpha v^{\alpha-1}) + e^{-tv^\alpha}\right]dt$$

$$= \int_0^\infty F(t)\,v\,e^{-tv^\alpha}\,(-tv\,v^{\alpha-1}) + \int_0^\infty f(t)\,e^{-tv^\alpha}dt$$

$$= \frac{1}{v}\left[v\int_0^\infty f(t)v\,e^{-tv^\alpha}(-t\alpha v^{\alpha-1})dt\right] + \frac{1}{v}\int_0^\infty F(t)e^{-tv^\alpha}dt$$

$$= -\alpha v^{\alpha-1}v\int_0^\infty F(t)t\,e^{-tv^\alpha}\,dt + \frac{1}{v}S(v)$$

$\therefore K[tf(t)] = \frac{-1}{\alpha v^{\alpha-1}}\left[\frac{d}{dv}\ s(v) - \frac{1}{v}\ S(v)\right]$

$\therefore K[tf(t)] = \frac{-1}{\alpha v^{\alpha-1}}\left[\frac{d}{dv}\ - \frac{1}{v}\right] S(v)$

Hence proved

2.4

4. CONVOLUTION THEOREM FOR KUSHARE TRANSFORM:

3.1. If Kushare transform of functions $f(t)$ and $g(t)$ are $f(v)$ and $g(v)$ respectively then Kushare Transform of their convolution $f(t)\&g(t)$ is given by,

$$K[f(t)*g(t)] = \frac{1}{v}f(v)\cdot g(v)$$

Proof: The convolution of function $f(t)\&g(t)$ is

$$(f*g)(t) = \int_0^t f(\tau)\ g(t-\tau)\ d\tau$$

Applying Kushare transform, we get

$$K[(f*g)(t)] = K\left[\int_0^t f(\tau)\ g(t-\tau)\ d\tau\right]$$

We know by definition of Kushare transformation

$$K[f(t)] = v\int_0^\infty f(t)e^{-tv^\alpha}dt$$

$$K[(f*g)(t)] = v\int_0^\infty e^{-tv^\alpha}\int_0^t f(\tau)g(t-\tau)d\tau dt$$

$$= v\int_0^\infty\int_\tau^\infty e^{-tv^\alpha}f(\tau)\ g(t-\tau)dtd\tau$$

Now, set $-\tau = b$, we get

$$K[(f*g)(t)] = v\int_0^\infty\int_0^\infty e^{-(\tau+b)v^\alpha}f(\tau)g(b)dbd\tau$$

$$= v\int_0^\infty\int_0^\infty e^{-\tau v^\alpha}e^{-bv^\alpha}f(\tau)g(b)dbd\tau$$

$$= \frac{1}{v}\left\{v\int_0^\infty e^{-\tau v^\alpha}f(\tau)\ d\tau . v\int_0^\infty e^{-bv^\alpha}g(b)db\right\}$$

$$K[f(t)*g(t)] = \frac{1}{v}f(v)\cdot g(v)$$

4.1. IMPORTANT PROPERTIES OF ERROR FUNCTION AND COMPLIMENTARY ERROR FUNCTION:

1. Theorem of error and complimentary error function is unity: $erf(x) + \operatorname{erf}c(x) = 1$

Proof: we have,

$$\int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2} \Rightarrow \frac{2}{\pi}\int e^{t^2}\ dt = 1$$

$\therefore \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\ dt + \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\ dt = 1 \Longrightarrow \operatorname{erf}x + \operatorname{erf}cx = 1$

2. Error function is an odd function:

$$\operatorname{erf}(-x) = -\operatorname{erf}(x)$$

3. The value of error function at $x = 0$ is 0

$$\operatorname{erf}(0) = 0$$

4. The value of complimentary error function at $x = 0$ is 1 then $\operatorname{erf}c(0) = 1$

5. The domain of error and complimentary error function is $(-\infty,\infty)$

6. $erf(x) \to 1$ as $x \to \infty$

7. $\operatorname{erf}c(x) \to 0$ as $x \to \infty$

8. The value of error function $\operatorname{erf}(x)$ for different values of x:

Table 1: Values of Complimentary error function at x= 0

1.	0.00	0.0000
2.	0.02	0.02256
3.	0.04	0.04511
4.	0.06	0.06762
5.	0.08	0.09008
6.	0.10	0.11246
7.	0.12	0.13476
8.	0.14	0.15695

5. KUSHARE TRANSFORM FOR ERROR FUNCTION:

We know by definition of error function

$$\operatorname{erf}\sqrt{t} = \frac{2}{\sqrt{\pi}}\int_0^t e^{x^2}dx$$

$$= \frac{2}{\sqrt{\pi}}\int_0^t\left[1 - x^2 + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right]dx$$

Applying Kushare Transform, we get

$$K[\operatorname{erf}\sqrt{t} =]\frac{2}{\sqrt{\pi}}\left[t^{1/2} - \frac{t^{3/2}}{1!\,3} + \frac{t^{5/2}}{2!\,.5} - \frac{t^{7/2}}{3!\,.7} + \cdots\right]$$

$= \frac{2}{\sqrt{\pi}}\left[\frac{1/2\,\Gamma 1/2}{v^{\alpha(3/2)-1}} - \frac{\Gamma 5/2}{v^{\alpha(5/2)-1}} + \frac{\Gamma 7/2}{v^{\alpha(7/2)-1}} - \cdots\right]$ ----------------------by Kushare transform

$$= \frac{2}{\sqrt{\pi}}\left[\frac{\sqrt{\pi}}{2\ v^{\alpha(3/2)-1}} - \frac{3\sqrt{\pi}}{4v^{\alpha(5/2)-1}} + \frac{15\sqrt{\pi}}{8v^{\alpha(7/2)-1}} - \cdots\right]$$

$$K[\operatorname{erf}\sqrt{t}] = \frac{1}{v^{\alpha(1/2)}} - \frac{3}{2}\frac{1}{v^{\alpha(3/2)}} + \frac{15}{4}\frac{1}{v^{\alpha(5/2)}} - \frac{105}{8}\frac{1}{v^{\alpha 7/2}} + \cdots$$

$$\therefore K[\operatorname{erf}\sqrt{t}] = \frac{1}{\sqrt{v^\alpha}}\frac{1}{\sqrt{1+\frac{1}{v^\alpha}}} = \frac{1}{\sqrt{v^\alpha+1}}.$$

6. APPLICATIONS OF KUSHARE TRANSFORM IN IMPROPER INTEGRALS:

In this section we solve some improper integrals.

Example 1: Evaluate the improper integral $I = \int_0^\infty e^{-t}\operatorname{erf}\sqrt{t}dt$.

Solution: we know,

$K[\operatorname{erf}\sqrt{t}] = \frac{1}{\sqrt{v^\alpha+1}}$________________ (1)

By the definition of Kushare transformation,

$$K[f(t)] = v\int_0^\infty f(t)e^{-tv^\alpha}dt$$

$K[\mathrm{erf}\sqrt{t}] = v\int_0^\infty \mathrm{erf}\sqrt{t}\, e^{-tv}\, dt$ ______________(2)

∴ From equation (1) & (2) , we get

$$v\int_0^\infty \mathrm{erf}\sqrt{t}\, e^{-tv}\, dt = \frac{1}{\sqrt{v^\alpha+1}}$$

Now put $v = 1$ & $\alpha = 1$, we get

$$\int_0^\infty \mathrm{erf}\sqrt{t}\, e^{-tv}\, dt = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{t}}$$

Hence,

$$\int_0^\infty \mathrm{erf}\sqrt{t}\, e^{-tv}\, dt = \frac{1}{\sqrt{2}}$$

Example 2: Evaluate the improper integral

$$I = \int_0^\infty t\, e^{-3t}\, \mathrm{erf}\sqrt{t} dt$$

Solution: We have,

$$K[\mathrm{erf}\sqrt{t}] = \frac{1}{\sqrt{v^\alpha+1}} = S(v)$$

&we also have,

$$K[tF(t)] = \left(\frac{1}{\alpha v^{\alpha-1}}\right)\left[\frac{d}{dv} - \frac{1}{v}\right] S(v)$$

$$\therefore K[t\,\mathrm{erf}\sqrt{t}] = \frac{-1}{\alpha v^{\alpha-1}}\left[\frac{d}{dv} - \frac{1}{v}\right]\frac{1}{\sqrt{v^\alpha+1}}$$

$$= \frac{-1}{\alpha v^{\alpha-1}}\left[\frac{d}{dv}\frac{1}{\sqrt{v^\alpha+1}} - \frac{1}{v}\frac{1}{\sqrt{v^\alpha+1}}\right]$$

$$= \frac{-1}{\alpha v^{\alpha-1}}\left[\left(\frac{-(\alpha v^{\alpha-1})}{2(v^\alpha+1)^{3/2}}\right) - \frac{1}{v\sqrt{v^\alpha+1}}\right]$$

$$= \frac{1}{2(v^\alpha+1)^{3/2}} + \frac{1}{v\sqrt{v^\alpha+1}}\frac{1}{\alpha v^{\alpha-1}}$$ ______________ (1)

Now, by the definition of Kushare Transformation, we have

$K[t\,\mathrm{erf}\sqrt{t}] = v\int_0^\infty t\,\mathrm{erf}\sqrt{t}\, e^{-tv^\alpha} dt$ ______________ (2)

∴From equation (1) & (2), we get

$$v\int_0^\infty t\,\mathrm{erf}\sqrt{t}\, e^{-tv^\alpha} dt = \frac{1}{2(v^\alpha+1)^{3/2}} + \frac{1}{v\sqrt{v^\alpha+1}}\frac{1}{\alpha v^{\alpha-1}}$$

Now, put $v = 3$ & $\alpha = 1$

$$3\int_0^\infty t\, erf\,\sqrt{t}\, e^{-3t} dt = \frac{1}{2(3+1)^{3/2}} + \frac{1}{3\sqrt{3+1}}.1$$

$$= \frac{1}{2(64)^{1/2}} + \frac{1}{3\times 2}$$

$$= \frac{1}{16} + \frac{1}{6}$$

$$3\int_0^\infty t\, erf\,\sqrt{t}\, e^{-3t} dt = \frac{6+16}{96}$$

$$= \frac{22}{96}$$

$$\int_0^\infty t\,\mathrm{erf}\sqrt{t}\, e^{-3t} dt = \frac{22}{96}\times\frac{1}{3} = \frac{22}{288}$$

$$\int_0^\infty t\,\mathrm{erf}\sqrt{t}\, e^{-3t} dt = \frac{11}{144}$$

Example 3: Evaluate the improper integral

$$I = \int_0^\infty e^{-5t}\{erf\,\sqrt{t} * erf\,\sqrt{t}\} dt$$

Solution: We have,

$$K[\mathrm{erf}\sqrt{t}] = \frac{1}{\sqrt{v^\alpha+1}} = S(v)$$

By the convolution theorem of Kushare Transformation, we have

$$K[f(t)*g(t)] = \frac{1}{v} f(v)\cdot g(v)$$

$$\therefore K[\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}] = \frac{1}{v} K[\mathrm{erf}\sqrt{t}]\cdot K[\mathrm{erf}\sqrt{t}]$$

$$\therefore K[\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}] = \frac{1}{v}\frac{1}{\sqrt{v^\alpha+1}}\cdot\frac{1}{\sqrt{v^\alpha+1}}$$

$$= \frac{1}{v}\frac{1}{v^\alpha+1}$$

Put, $\alpha = 1$

$K[\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}] = \frac{1}{v(v+1)}$ ______________ (1)

Now, by definition of Kushare Transform,

$$K[\mathrm{erf}\sqrt{t} * \mathrm{erf}\sqrt{t}] = v\int_0^\infty e^{-tv^\alpha}[erf\,\sqrt{t} * erf\,\sqrt{t}] dt$$

∴ From (1) & (2), we get

$$v\int_0^\infty e^{-tv^\alpha}[erf\,\sqrt{t} * erf\,\sqrt{t}] dt = \frac{1}{v}\frac{1}{v^\alpha+1}$$

Now put, $= 5$,$\alpha = 1$, we get

$$5\int_0^\infty e^{-t5^1}[erf\,\sqrt{t} * erf\,\sqrt{t}] dt = \frac{1}{5(5+1)}$$

$$= \frac{1}{30}$$

$$\int_0^\infty e^{-t5^1}[erf\,\sqrt{t} * erf\,\sqrt{t}] dt = \frac{1}{5}\cdot\frac{1}{30}$$

$\therefore I = \frac{1}{150}$

7. Conclusion:

In this article , we have successfully discussed the KUSHARE TRANSFORM of Error function .The given numerical application section show the use of KUSHARE TRANSFORM of error function for evaluating improper integral, which contain error function . Results of numerical applications show KUSHARE TRANSFORM gives the exact solution without any tedious calculation work.

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