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Thermal radiation effect on an unsteady hydromagnetic free convective oscillatory Couette flow of a viscous fluid embedded in a porous medium

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Abstract: ***This paper investigates the effect of thermal radiation on an unsteady magnetohydrodynamic free convective oscillatory Couette flow of an optically, viscous thin fluid bounded by two horizontal porous parallel walls under the influence of an externalimposed transverse magnetic field embedded in a porous medium.The fluid is considered to be a gray, absorbing – emitting but non – scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The non – dimensional governing coupled equations involved in the present analysis are solved by an efficient, accurate, and extensively validated and unconditionally stable finite difference scheme of the Crank Nicolson method and the expressions for velocity, temperature, Skin friction and rate of heat transfer has been obtained.Numerical results for velocity and temperatureare presented graphically and the numerical values of Skin friction and Nusselt numberhave been tabulated.The effect of different parameters like thermal Grashof number, Magnetic field (Hartmann number), Prandtl number, Porosity parameter and Thermal radiation parameteron the velocity, temperature, Skin friction and Nusselt number are discussed.***

Keywords: Thermal radiation, MHD, Free convection, Couette flow, Periodic wall temperature and Finite difference method.

1. Introduction

Magnetohydrodynamic (MHD) free convection of a viscous incompressible fluid along a vertical wall in porous medium must be studied if we are to understand the behaviour of fluid motion in many applications as for example, in MHD electrical power generation, geophysics, astrophysics, etc. The problem of free convection flows of viscous incompressible fluids past a semi – infinite vertical wall has received a great deal of attention in recent years because of its many practical applications, such as in electronic components, chemical processing equipment, etc. Magneto hydrodynamic free convection flow of an electrically conducting fluid in different porous geometries is of considerable interest to the technical field due to its frequent occurrence in industrial, technological and geothermal applications. As an example, the geothermal region gases are electrically conducting and undergo the influence of magnetic field. Also, it has applications in nuclear engineering in connection with reactors cooling. The interest in this field is due to the wide range of applications in engineering and geophysics, such as the optimization of the solidification processes of metals and metal alloys, the study of geothermal sources, the treatment of nuclear fuel debris, the control of underground spreading of chemical wastes and pollutants and the design of MHD power generators. Many papers concerned with the problem of MHD free convection flow in porous media have been published in the literature.

In view of its wide applications, the unsteady free convective MHD flow with heat transfer past a semi – infinite vertical porous moving plate with variable suction has been studied by Kim [1]. Singh and Thakur [2] have given an exact solution of a plane unsteady MHD flow of a non – Newtonian fluid. Sharma and Pareek[3] explained the behaviour of steady free convective MHD flow past a vertical porous moving surface. Singh and his co – workers [4] have analysed the effect of heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Makindeet al.[5] discussed the unsteady free convective flow with suction on an accelerating porous plate. Sarangi and Jose [6] studied the unsteady free convective MHD flow and mass transfer past a vertical porous plate with variable temperature. Das and his associates [7] estimated the mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction employing finite difference analysis. Das et al.[8] investigated numerically the unsteady free convective MHD flow past an accelerated vertical plate with suction and heat flux. Das and Mitra [9] discussed the unsteady mixed convective MHD flow and mass transfer past an accelerated infinite vertical plate with suction. Recently, Das and his co – workers [10] analyzed the effect of masstransfer on MHD flow and heat transfer past avertical porous plate through a porous medium under oscillatory suction and heat source. GireeshKumar et al.[11] investigated effects of chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink.

The effect of radiation is quite significant at high temperature. Radiative convective flows are encountered in countless industrial and environment processes, particularly in astrophysical studies and space technology.

Radiative heat and mass transfer play an importantrole in manufacturing industries for the design of fins,steel rolling, nuclear power plants, gas turbines andvarious propulsion device for aircraft, combustion andfurnace design, materials processing, energy utilization,temperature measurements, remote sensing forastronomy and space exploration, food processing andcryogenic engineering, as well as numerous agricultural,health and military applications.Arpaci [12] studied the interaction between thermal radiation and laminar convection of heated vertical plate in a stagnant radiating gas. England and Emery [13] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. In all above studies, the stationary plate was considered. Singh [14] studied the effects of coriolis as well as magnetic force on the flow field of an electrically conducting fluid past an impulsively started infinite vertical plate. Bestman and Adjepong [15] studied the magnetohydrodynamic free convection flow, with radiative heat transfer, past an infinite moving plate in rotating incompressible, viscous and optically transparent medium. Das et al. [16] have analysed radiation effects on flow past an impulsively started infinite isothermal vertical plate. Raptis and Perdikis [17] considered the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically.

The convection problem in a porousmedium has important applications ingeothermal reservoirs and geothermalextractions. The process of heat and masstransfer is encountered in aeronautics, fluidfuel nuclear reactor, chemical processindustries and many engineeringapplications in which the fluid is theworking medium. The wide range oftechnological and industrial applications hasstimulated considerable amount of interest inthe study of heat and mass transfer inconvection flows. The radiation effect onheat transfer over a stretching surface hasbeen studied by Elbashbeshy [18]. Takharet al. [19] studied the radiation effects on MHD freeconvectionflow of a gas past a semi-infinitevertical plate. Thermal radiation andbuoyancy effects on MHD free convectiveheat generating flow over an acceleratingpermeable surface with temperaturedependent viscosity has been studied bySeddeek [20]. Ghaly and Elbarbary [21] haveinvestigated the radiation effect on MHDfree convection flow of a gas at a stretchingsurface with a uniform free stream. In all theabove studies, only steadystate flows oversemi – infinite vertical plate have been considered. The unsteady freeconvectionflows over a vertical plate has been studiedby Gokhale [22] and Muthucumaraswamy and Ganesan [23]. Cheng andTeckchandani [24] obtained numerical solutionsfor the convective flow in a porous mediumbounded by two isothermal parallel plates inthe presence of the withdrawal of the fluid.In all the above mentioned studies, the effectof porosity, permeability and the thermalresistance of the medium is ignored ortreated as constant. However, porositymeasurements by Benenati and Broselow [25] show that porosity is not constant but variesfrom the surface of the plate to its interior towhich as a result permeability also varies. Incase of unsteady free convective flows, the combined effect ofbuoyancy forces from thermal and massdiffusion on forced convection was studiedby Chen et al. [26]. Bejan and Khair [27] haveinvestigated the vertical free convectiveboundary layer flow embedded in a porousmedium resulting from the combined heatand mass transfer. Lin and Wu [28] wereanalysed the problem of simultaneous heatand mass transfer with the entire range ofbuoyancy ratio for most practical andchemical species in dilute and aqueoussolutions. Rushi Kumar and Nagarajan [29] studied the mass transfer effects of MHDfree convection flow of an incompressibleviscous dissipative fluid past an infinitevertical plate. Mass transfer effects on

Freeconvection flow of an incompressibleviscous dissipative fluid have been studiedby Manohar and Nagarajan [30].

In fluid dynamics, Couette flow refers to the free convection flow of a viscous fluid in the space between two parallel plates, one of which moving relative to the other. The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates. This type of flow is named in honour of Maurice Marie Alfred Couette, a professor of physics at the French university of Angers in the late 19thcentury. Couette flow is frequently used in undergraduate physics and engineering courses to illustrate shear – driven fluid motion. Some important application areas of Couette motion are megnetohydrodynamics power generators and pumps, polymer technology, petroleum industry and purification of crude oil etc. An analysis of flow of Couette flow has been studied extensively for the case of horizontal channel. Choi et al. [31] studied the buoyancy effects in plane Couette flow heated uniformly from below with constant heat flux. Attia and Sayed – Ahmed [32] investigated the problem of the effect Hall currents on unsteady MHD Couette flow and heat transfer of a Bingham fluid with suction and injection. The effectiveness of variation in the physical variables on the generalized Couette flow with heat transfer in a porous mediumstudied by Attia [33]. Makinde and Osalusi [34] considered the problem of MHD steady flow in a channel filled with porous material with slip at the boundaries, while,Narahari [35] studied the effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary. Israel – Cookeyet al. [36]discussed oscillatory magnetohydrodynamic Couette flow of a radiating viscous fluid in a porous medium with periodic wall temperature.

The object of the present paper is to analyse the effect of thermal radiation on an unsteady magnetohydrodynamic free convective oscillatory Couette flow of an optically, viscous thin fluid bounded by two horizontal porous parallel walls under the influence of an externalimposed transverse magnetic field embedded in a porous medium.The fluid is considered to be a grey, absorbing – emitting but non – scattering medium and the Rosseland

approximation is used to describe the radiative heat flux in the energy equation.The non – dimensional governing coupled equations involved in the present analysis are solved by an efficient, accurate, and extensively validated and unconditionally stable finite difference scheme of the Crank Nicolson method which is more economical from computational view point.The effects of various governing parameters on the velocity, temperature, skin friction coefficient and Nusselt number are shown in figures and tables and discussed in detail. From computational point of view it is identified and proved beyond all doubts that the Crank Nicolson methods more economical in arriving at the solution and the results obtained
are good agreement with the results of Israel – Cookeyet al. [36]in some special cases.In section 2, the mathematical formulation of the problem and dimension less forms of the governing equations are established. Solution method tothese equations for the flow variables are briefly examined in section 3. Theresults of the previous sections are discussed in section 4. In section 5, generalconcluding remarks of the results of the previous sections are given.

2. Mathematical formulation

We consider theunsteady Couette flow of an electrically conductingand optically thin viscous incompressible fluid in aporous medium bounded by two infinite non – conducting horizontal parallel walls under theinfluence of an externally applied uniform magneticfield and radiative heat flux.

We made the following assumptions:

1. The axis is taken along the plate in the vertical upward direction and the axis is taken normal to the plate.
2. It is assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small, so we neglect the induced magnetic field.
3. The lower wall is suddenly moved from rest with a free stream velocity, that oscillates in time about a constant mean velocity .
4. It is assumed that the temperature of the moving lower plate oscillates in time about a non – zero constant mean.
5. The induced magnetic field, Hall Effect and viscous dissipation are assumed to be negligible as the magnetic Reynolds number of the flow is taken to be very small.
6. It is assumed that there is no applied voltage which implies the absence of an electric field.
7. Assuming Boussinesq's approximation for an incompressible fluid model.

The governing equations of the flow for an optically thin medium are

Momentum Equation:

$$\rho\frac{\partial u'}{\partial t'}=\rho\frac{dU'}{dt'}+\nu\frac{\partial^2 u'}{\partial y'^2}+\rho g\beta(T'-T_h')-\left(\frac{1}{K'}+\sigma B_o^2\right)(u'-U')\quad(1)$$

Energy Equation:

$$\rho C_p\frac{\partial T'}{\partial t'}=\kappa\frac{\partial^2 T'}{\partial y'^2}+\nu\left(\frac{\partial u'}{\partial y'}\right)^2-\frac{\partial q_r}{\partial y}\quad(2)$$

Where u' is the flow velocity in $x'-$direction, U' is the free stream velocity, U_o is the mean constant free stream velocity, ν is the viscosity, ρ is the fluid density, g is the acceleration due to gravity, β is the coefficient of thermal expansion, σ is the electric conductivity of the fluid, K' is the permeability of the porous medium, κ is the thermal conductivity, C_p is the specific heat capacity at constant pressure, T' is the fluid temperature, T_w' is the temperature of the upper wall, T_h' is the temperature of the lower wall and q_r is the radiative heat flux. Also, ω' is the frequency of oscillation, $\varepsilon << 1$ is a small parameter and $U'(t')=1+\varepsilon e^{i\omega' t'}$ is the freestream velocity.

The corresponding boundary conditions are

$$\left.\begin{aligned}&u'=U_o\left(1+\varepsilon e^{i\omega' t'}\right),\ T'=T_w'+\varepsilon(T_w'-T_h')e^{i\omega' t'}\ on\ y'=0\\&u'=0,\ T'=T_h'\ on\ y'=h\end{aligned}\right\}\quad(3)$$

The radiative heat flux term is simplified by making use of the Rosseland approximation [37] as

$$q_r=-\frac{4\bar{\sigma}}{3k^*}\frac{\partial T'^4}{\partial y'}\quad(4)$$

Here $\bar{\sigma}$ is Stefan – Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T_h' and neglecting higher – order terms. This results in the following approximation:

$$T'^4\cong 4T_h'^3T'-3T_h'^4\quad(5)$$

Using equations (4) and (5) in the last term of equation (2), we obtain:

$$q_r=-\frac{16\bar{\sigma}T_h'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2}\quad(6)$$

Introducing (6) in the equation (2), the energy equation becomes:

$$\rho C_p\frac{\partial T'}{\partial t'}=\kappa\frac{\partial^2 T'}{\partial y'^2}+\frac{16\bar{\sigma}T_h'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2}+\nu\left(\frac{\partial u'}{\partial y'}\right)^2\quad(7)$$

Introducing the following non – dimensional quantities into the equations (1) and (7):

$$\left.\begin{aligned} & y=\frac{y'}{h},\ u=\frac{u'}{U_o},\ U=\frac{U'}{U_o},\ t=\omega' t',\ \omega=\frac{\omega' h^2}{\nu},\ \theta=\frac{T'-T'_h}{T'_w-T'_h},\ Gr=\frac{g\beta h^2(T'_w-T'_h)}{\nu U_o}, \\ & \zeta^2=\frac{h^2}{K'},\ M^2=\frac{\sigma B_o^2 h^2}{\nu\rho},\ R=\frac{\kappa k^*}{4\sigma T_h'^3},\ \Pr=\frac{\rho\nu C_p}{\kappa},\ Ec=\frac{U_o^2}{C_p(T'_w-T'_h)},\ U(t)=1+\varepsilon e^{it} \end{aligned}\right\} \quad (8)$$

Then the equations (1) and (7), using non – dimensional quantities (8) reduce to the following non – dimensional form of equations: (8)

$$\omega\frac{\partial u}{\partial t}=\omega\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial y^2}+(Gr)\theta-(\zeta^2+M^2)(u-U) \quad (9)$$

$$\omega(\Pr)\frac{\partial\theta}{\partial t}=\left(\frac{3R+4}{3R}\right)\frac{\partial^2\theta}{\partial y^2}+(\Pr)(Ec)\left(\frac{\partial u}{\partial y}\right)^2 \quad (10)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} & u=1+\varepsilon e^{it},\ \theta=1+\varepsilon e^{it}\ \ on\ \ y=0 \\ & u\to 0,\ \theta\to 0\ \ on\ \ y=1 \end{aligned}\right\} \quad (11)$$

For practical engineering applications and the design of chemical engineering systems, quantities of interest include the following Skin friction, Nusselt number and Sherwood number are useful to compute. This is given by which is written in dimensionless form as

$$\tau=\frac{\tau_w}{\rho u_w^2},\ \tau_w=-\left[\nu\frac{\partial u}{\partial y}\right]_{y'=0}=-\rho U_o^2 u'(0)=-\left[\frac{\partial u}{\partial y}\right]_{y=0}$$

(12)

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$N_u(x')=-\left[\frac{x'}{(T'_w-T'_h)}\frac{\partial T'}{\partial y'}\right]_{y'=0} \quad \text{then}$$

$$Nu=\frac{N_u(x')}{R_{e_x}}=-\left[\frac{\partial\theta}{\partial y}\right]_{y=0} \quad (13)$$

These two are calculated by numerical differentiation using Newton's forward Interpolation formula. During computation of the above quantities, the non – dimensional time is fixed at $t=1.0$.The mathematical formulation of the problem is now completed. Equations (9)& (10)present a coupled nonlinear system of partial differential equations and are to be solved by using initial and boundary conditions (11). However, exact solutions are difficult, whenever possible. Hence, these equations are solved by the Crank Nicholson method.

2.Numerical Solution by Crank Nicholson Method:

Equations (9) & (10) represent coupled system of non – linear partial differential equations which are solved numerically under the initial and boundary conditions (11) using the finite difference approximations. A linearization technique is first applied to replace the non – linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank Nicolson implicit method is used at two successive time levels [38]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas – algorithm [38]. Finite difference equations relating the variables are obtained by writing the equations at the midpoint of the computational cell and then replacing the different terms by their second order central difference approximations in the direction. The diffusion terms are replaced by the average of the central differences at two successive time – levels. The computational domain is divided into meshes of dimension Δt and Δy in time and space respectively as shown in figure 1. We define the variables $B=u_y$ and $L=\theta_y$ to reduce the second order differential equations (9) &(10) to first order differential equations. The finite difference representations for theresulting first order differential equations (9)& (10) take the following forms:

$$\begin{aligned} & \omega\left(\frac{u_{i+1,j+1}-u_{i,j+1}+u_{i+1,j}-u_{i,j}}{2(\Delta t)}\right)=\omega(i\varepsilon e^{it})+\left(\frac{(B_{i+1,j+1}+B_{i,j+1})-(B_{i+1,j}+B_{i,j})}{2(\Delta y)}\right)+ \\ & Gr\left(\frac{\theta_{i+1,j+1}+\theta_{i,j+1}+\theta_{i+1,j}+\theta_{i,j}}{4}\right)-(\zeta^2+M^2)\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{4}-U\right) \end{aligned} \quad (14)$$

$$\omega\left(\frac{\theta_{i+1,j+1}-\theta_{i,j+1}+\theta_{i+1,j}-\theta_{i,j}}{2(\Delta t)}\right)=\frac{1}{\Pr}\left(\frac{3R+4}{3R}\right)\left(\frac{(L_{i+1,j+1}+L_{i,j+1})-(L_{i+1,j}+L_{i,j})}{4}\right)+QZFO \quad (15)$$

Where represents the viscous dissipation term which are known from the solution of the momentum equations and can be evaluated at the midpoint of the computational cell. Computations have been made for 2.0, 2.0, 0.71, 2.0, 2.0, 0.001, 5.0 and 1.0. Grid – independence studies show that the computationaldomain and can be divided intointervals with step sizes 0.0001 and 0.005 fortime and space respectively. The truncation error of thecentral difference schemes of the governing equations is . Stability and rate of convergence are functions of the flow and heat parameters. Smaller step sizesdo not show any significant change in the results. Convergenceof the scheme is assumed when all of the unknowns and for the last two approximations differ from unity by less than for all values of in at every time step. Less than 7 approximationsare required to satisfy this convergence criteria forall ranges of the parameters studied here.

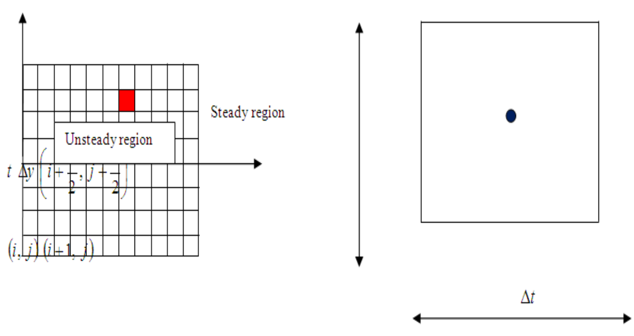

Figure 1. Mesh Layout

3. Results and Discussion:

The problem of MHD free convection and oscillatoryflow of an optically thin fluid bounded by two parallelwalls under the influence of an external imposedtransverse magnetic field in a porous medium hasbeen studied. By taking the radiative heat flux in thedifferential form and imposing an oscillatory timedependenton the coupled non – linearproblem is solved for the velocity and temperatureprofiles. In order to understand the physical situationof the problem and hence the manifestations of thevarious material parameters entering the problem wehave computed the numerical values of the velocity,temperature, skin friction and the Nusselt number using the software “Mat – Lab”.For the purposesof our computation, we set 2.0, 2.0, 2.0, 2.0, 0.001, 5.0 and 1.0 and Prandtl number, 0.71whichphysically corresponds to the atmosphericenvironment (air) at fixed for the velocity andtemperature profiles.The influence of the thermal Grashof number Gr on the velocity is presented in figure (2). The thermal Grashof number signifies the relative effectof the thermal buoyancy force to the viscoushydrodynamic force in the boundary layer. Asexpected, it is observed that there is a rise in thevelocity due to the enhancement of thermalbuoyancy force. Also, asGrincreases, the peakvalues of the velocity increases rapidly near theporous plate and then decays smoothly to the free stream velocity. Figure (3) illustrate the influences of Hartmann number M on the velocity field. It is found that the velocity decreases with increasing Hartmann number for air (Pr = 0.71) in presence of Hydrogen. The presence of transverse magnetic field produces a resistive force on the fluid flow. This force is called the Lorentz force, which leads to slow down the motion of electrically conducting fluid.

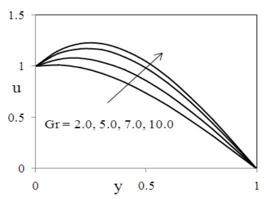

Figure 2. Velocity profiles forvariations in Grashof number

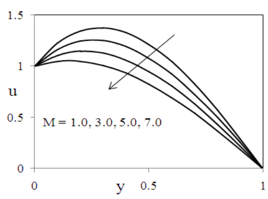

Figure 3. Velocity profiles forvariations in Hartmann number

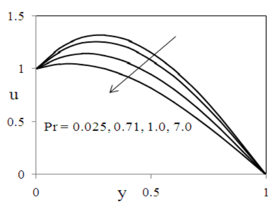

Figure 4. Velocity profiles for variations in Prandtl number

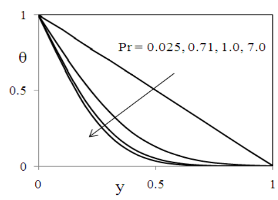

Figure 5. Temperature profiles for variations in Prandtl number

Figures (4) and (5) illustrate the velocity and temperature profiles for different values of the Prandtl numberPr. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical resultsshow that theeffect of increasing values of Prandtl numberresults in a decreasing velocity (figure (4)). From figure (5), it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Prare equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values ofPr. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

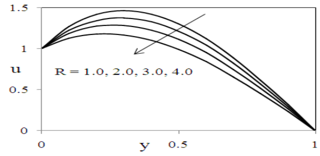

Figure 6. Velocity profiles for variations in thermal Radiation parameter

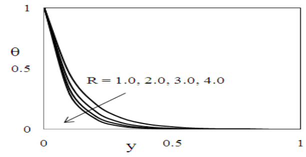

Figure 7. Temperature profiles for variations in Thermal radiation parameter

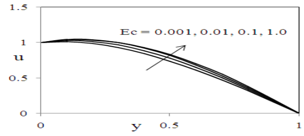

Figure 8. Velocity profiles for variations in Eckert number

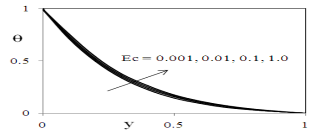

Figure 9. Temperature profiles forvariations in Eckert number

The effects of the thermal radiation parameter (R) on the velocity and temperature profiles in the boundary layer are illustrated in figures (6) and (7) respectively. Increasing the thermal radiation parameter (R) produces significant increase in the thermal condition of the fluid and its thermal boundary layer. This increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase.The influence of the viscous dissipation parameter i.e., the Eckert number (Ec) on the velocity and temperature are shown in figures (8) and (9) respectively. The Eckert number (Ec) expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature as well as the velocity. This behaviour is evident from figures (8) and (9). Figure (10) shows the velocity profiles for different values of the Darcy number (ζ). Clearly ζas increases the peak value of velocity tends to decrease

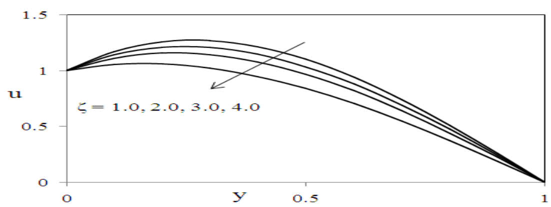

Figure 10. Velocity profiles for variations in Porosity parameter

Table – 1:Skin friction (τ) values for different values of Gr, M, Pr, ζ, R and Ec

Gr	M	Pr	ζ	R	Ec	τ
1.0	1.0	0.71	1.0	1.0	0.001	1.8726
2.0	1.0	0.71	1.0	1.0	0.001	2.9303
1.0	2.0	0.71	1.0	1.0	0.001	0.9876
1.0	1.0	7.00	1.0	1.0	0.001	1.5504
1.0	1.0	0.71	2.0	1.0	0.001	1.3877
1.0	1.0	0.71	1.0	2.0	0.001	1.1165
1.0	1.0	0.71	1.0	1.0	0.100	1.9413

The profiles for Skin friction due to velocity under the effects of Grashof number, Hartmann number, Prandtl number, Porosity parameter, Thermal radiation parameter and Eckert number are presented in the table – 1. We observe from this table – 1 the Skin friction due to velocity rises under the effects of Grashof number and Eckert number and falls under the effects of Hartmann number, Prandtl number, Porosity parameter and Thermal radiation parameter.

Table – 2: Rate of heat transfer (Nu) values for different values of Pr, Ec and R

Pr	Ec	R	Nu
0.71	0.001	1.0	1.7282
7.00	0.001	1.0	0.9924
0.71	0.100	1.0	1.9287
0.71	0.001	2.0	1.0642

The profiles for Nusselt number (Nu) due to temperature profile under the effect of Prandtl number, Thermal radiation parameter and Eckert number are presented in the table– 2. We see from this table – 2, the Nusselt number due to temperature profile rises under the effect of Eckert numberand falls under the effect of Prandtl number and Thermal radiation parameter.In order to ascertain the accuracy of the numerical results, the present results are compared with the existed results of Israel – Cookey *et al.* [36] for $Gr = 2.0$, $\zeta = 2.0$, $R = 2.0$, $M = 2.0$, $\omega = 5.0$, $t = 1.0$ and $\Pr = 0.71$ are presented in table – 4. They are found to be in an excellent agreement.

Table – 4: Comparison of present Skin friction results (τ) with the Skin friction results(τ^*) obtained by Israel – Cookey *et al.* [36] for different values of Gr, ζ, R, M and Pr

Gr	M	Pr	ζ	R	τ	τ^*
1.0	1.0	0.71	1.0	1.0	1.7842	1.7840
5.0	1.0	0.71	1.0	1.0	2.2148	2.2144
1.0	2.0	0.71	1.0	1.0	1.4059	1.4051
1.0	1.0	7.00	1.0	1.0	1.5105	1.4995
1.0	1.0	0.71	2.0	1.0	1.5391	1.5377
1.0	1.0	0.71	1.0	2.0	1.4982	1.4978

4. Conclusions

In conclusion therefore, the effect of thermal radiation on an unsteady magnetohydrodynamic free convective oscillatory Couette flow of an optically, viscous thin fluid bounded by two horizontal porous parallel walls under the influence of an external imposed transverse magnetic field embedded in a porous medium. The non – dimensional governing coupled equations involved in the present analysis are solved by finite difference scheme of the Crank Nicolson method and the expressions for velocity, temperature, Skin friction and rate of heat transfer has been obtained. The results are presented graphically and we can conclude that the flow field and the quantities of physical interest are significantly influenced by these parameters.

1. Grashof number and Eckert number tend to retard fluid velocity whereas Hartmann number, Prandtl number, Porosity number and Thermal radiation parameter have reverse effect on it.
2. Eckert number tends to enhance fluid temperature whereas Prandtl number and Thermal radiation parameter have reverse effect on it.
3. The profiles for Skin frictiondue to velocity profiles rises under the effects of Grashof number and Eckert number and falls under the effects of Prandtl number, Porosity number and Thermal radiation parameter.
4. The profiles for Nusselt numberdue to temperature profiles rises under the effect of Eckert number and falls under the effects of Prandtl number and Thermal radiation parameter.
5. On comparing the Skin friction results with the results of Israel – Cookey et al. [36] it can be seen that they agree very well.

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