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A Survey of Graph Matching Techniques Using Quantum Walks

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Abstract
Graph matching techniques have been projected as a powerful tool for Pattern Recognition since the inception of machine vision. The reason might be the fact that the computational cost of the graph-based algorithms, although still high in most cases, is now becoming compatible with high performance computing. Here we endeavour to catalogue the literature on quantum walk techniques for graph matching. A continuous time quantum walk consists of a particle moving among the vertices of a graph which cannot distinguish two non-isomorphic graphs with high symmetry. We then discuss the quantum walks involving more than one particle. In addition to moving around the graph, these particles interact when they come across one another. Our focus is on a class of graphs with specifically high symmetry called strongly regular graphs. Our goal in this paper is to deliver a theoretical association of graph matching techniques using quantum walks.

Keywords: graph isomorphism, quantum walks, continuous-time quantum walk, interacting bosons.

1. Introduction
In many applications a fundamental operation is the comparison between two objects or between an object and a model to which the object could be associated. The graph is an intelligent illustration of any structured object in science and engineering. To date, the best known classical algorithm has a runtime of, where c is a constant and N is the number of vertices in the graph [1]. GI is thought to be similar to integer factoring [2] in that both could reside in the complexity class NP-Intermediate. Given that there exists a polynomial-time quantum algorithm for integer factoring, it is thought that there may exist a quantum speedup for GI as well. Random walks and kernels are used to solve GI problem. A new class of algorithms that has been explored for GI is quantum random walks. Quantum random walks are gaining popularity they are known to provide computational advantages over classical random walks [3]. The quantum random walks are faster than their fastest known classical analogues. Two types of quantum walks, discrete and continuous time, are introduced as the quantum walks for the corresponding random walks and have been discovered over the last few years.

2. Literature Review

2.1 Graph isomorphism
Gold and Rangarajan [4] have exploited the stochastic properties of Sinkhorn matrices to recover the matches using a softassign update algorithm. Umeyama [5] takes a more conventional least-squares approach and shows how an Eigen decomposition method can be used to recover the permutation matrix. Edwin R. Hancock [6] have used a graph-editing technique for accommodating structural inexactness and [7] have used the EM algorithm and Cast the recovery of correspondence matches between the graph-nodes in a matrix framework. They have accurately match inexact graphs under considerable levels of structural corruption using singular value decomposition to estimate the correspondence indicators. Xiao Bai, Hang Yu, and Edwin R. Hancock [8] used ISOMAP algorithm and were able to overcome problems of structural difference and [9] have used Young Householder decomposition on Heat Kernel.

2.2. Quantum Walks
Quantum computing has recently attracted attention because of the potential for considerable speed-ups over classical algorithms. For instance, Grover's search algorithm [10] is quadratically faster and Shor's factorization algorithm [11] is exponentially faster than known classical algorithms. These have been introduced as quantum counterparts of random walks and a good summary is given by Kempe [12]. The behavior of quantum walks is governed by unitary rather than stochastic matrices. A unitary matrix, on the other hand, has complex entries. Quantum walks possess a number of interesting properties not exhibited by classical random walks. For instance, because the evolution of the quantum walk is unitary and therefore reversible, the walks are non-ergodic and they do not have a limiting distribution.
There are two different models for the quantum random walk, both of which can be simulated on an arbitrary graph. The first of these is the continuous-time quantum walk proposed by Fahri and Gutmann [13]. They took the vertices of the graph as the basis vectors of the Hilbert space in which the walk takes place. It is necessary to define a Hamiltonian, H, which gives the energy for each possible transition. The evolution of the walk is given by Schrödinger's equation. One approach is that proposed by Gudkov et. al. [14] who suggested modeling the graph as a set of point particles with an attractive force between

adjacent vertices. The quantum walk system is evolved from a set initial state and the set of ordered separation distances calculated. This system solves the graph isomorphism problem. However, Shiau [15] show that it doesn't fail to distinguish between any pair of strongly regular graphs with the same set of parameters. They build on Gudkov et al's approach using the ideas of Rudolph [16] to construct graph invariants in a similar way, but based on the random walks generated by two particles on the graphs obeying Fermi statistics. An exponential speed-up for the hitting time was also observed by Kempe [12] for the discrete-time quantum walk. The quantum version of the discrete-time quantum walk on an arbitrary graph was formalized by Aharonov [18]. Kempe considered the walk on the n-dimensional hypercube and was able to show that the hitting time from one vertex to the one opposite is polynomial in n [17]. By making use of the exponentially faster hitting times that are observed for continuous-time quantum walks on graphs [19][20].

2.3. Single-particle Quantum Walks

Rudolph mapped the GI problem onto a system of interacting qubits [16]. One atom was used per vertex, and atoms i and j interacted if vertices i and j were connected by edges. He showed that graphs with same adjacency matrices may be non-isomorphic graphs because of the induced adjacency matrices describing transitions between three-particle states have different eigenvalues. Shiau et al. proved that the simplest classical algorithm fails to distinguish some pairs of non-isomorphic graphs [15] and also proved that continuous-time one-particle QRWs cannot distinguish some non-isomorphic graphs. Douglas and Wang modified a single-particle QRW by adding phase inhomogeneities, altering the evolution as the particle walked through the graph [21]. Most recently, Emms et al. used QRWs to build potential graph invariants [22]. Through numerical spectral analysis, they found that graph invariants could be used to distinguish many types of graphs by breaking the eigenvalue degeneracies of many families of graphs.

2.4. Two-particle Quantum Walks

In addition to studying single-particle QRWs, Shiau et al. [15] studied two- particle QRWs and presented proof that interacting Bosons can distinguish non-isomorphic pairs that single- particle walks cannot. There, it was also found numerically that two-Boson QRWs with non-interacting particles do not distinguish some non-isomorphic pairs of graphs. Berry and Wang [23] have compared the discrete time quantum walks and continuous time quantum walks. Their results indicate that discrete-time quantum random walks may have greater ability to distinguish non-isomorphic strongly regular graphs than continuous-time quantum random walks. M. A. Jafarizadeh [25] developed the quantum algorithms for distinguishing some non-isomorphic pairs of SRGs, by using the elements of blocks of adjacency matrices in the stratification basis. J. K. Gamble [24] proved that quantum walks of two non-interacting particles, Fermions or Bosons, cannot distinguish certain pairs of non-isomorphic SRGs. They have demonstrated numerically that two interacting Bosons are more powerful than single particles and two non-interacting particles, in that quantum walks of interacting bosons distinguish all non-isomorphic pairs of SRGs with up to 64 vertices. By performing a short-time expansion of the evolution operator, they derived distinguishing operators that provide analytic insight into the power of the interacting two-particle quantum walk.

3. BACKGROUND

3.1 Quantum walks on graphs

A graph G = (V, E) is a set vertices V and edges E. The vertices are usually labeled by integer indices, and the edges are unordered pairs of vertices. Two graphs are isomorphic if they can be made identical by relabeling their vertices. Graphs are conveniently expressed algebraically as adjacency matrices. The spectrum of a graph is the eigenvalue spectrum of its adjacency matrix. If we restrict ourselves to single-particle states, we find matrix elements

$$\langle i|H|j\rangle = -A_{ij} \quad \textbf{(1)}$$

Hence, we can easily identify the a single-particle Hamiltonian

$$H_{1P} = -A \quad \textbf{(2)}$$

3.2 Single-particle Quantum walks on graphs

D. Emms [22] has proposed a model with auxiliary graph fig. [1] where, $G=(V_G,E_G)$and $H=(V_H,E_H)$ then $\Gamma(G,H)=(V_\Gamma,E_\Gamma)$ and the vertex and edge sets can be decomposed such that $V_\Gamma = V_G \cup V_H \cup V_A$ and, $\varepsilon_\Gamma = \varepsilon_G G \cup \varepsilon_H \cup$

Where

$$A = \{v_{gi,hj} | g_i \in G, h_j \in H\}$$

and

$$A = \{g_i, v_{gi,hj}, h_j, v_{gi,hj} | g_i \in G, h_j \in H\}$$

The auxiliary vertices, A, serve to connect the two graphs and act as sites on which the interference takes place.

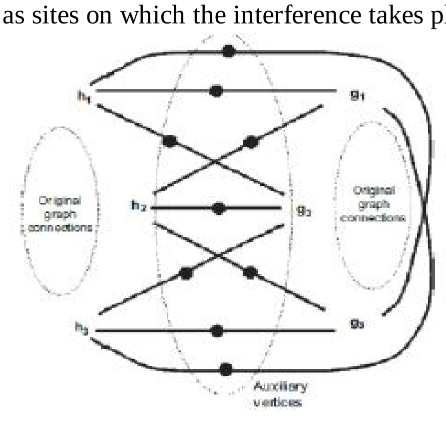

Figure 1: The auxiliary graph. [22].

The state of the quantum walk on Γ at time t is given by

$$|\Psi> = \sum_{u \in V} \alpha_u(t)|u> \quad (3)$$

The starting state has amplitudes

$$\alpha_u(0) = \begin{cases} \frac{d(u)}{C} & if\ u \in G \\ \frac{-d(u)}{C} & if\ u \in H \\ 0 & otherwise \end{cases} \quad (4)$$

where

$$C = \sum_{u \in V_G \cup V_H} d(u)^2 \quad (5)$$

is the normalization constant. The starting state for the walk on Γ is

$$|\Psi_0\rangle = \frac{1}{C}\left(\sum_{u \in V_G} |u\rangle - \sum_{v \in V_H} |v\rangle\right) \quad (6)$$

and

$$|\Psi_t\rangle = e^{-iLt}|\Psi_0\rangle \quad (7)$$

where L=D-A is the Laplacian matrix, A is the adjacency matrix and D is the diagonal degree matrix. We can use eigendecomposition of the Laplacian $L = \Phi^T \Lambda \Phi$, where Λ is the diagonal matrix with the ordered eigenvalues as elements and Φ is the eigenvector matrix with the respective ordered eigenvectors as columns.

$$\exp[-iLt] = \Phi^T \exp[-i\Lambda t]\Phi \quad (8)$$

Thus interference amplitude at vertex can be calculated as

$$\alpha_u(t) = \Phi^T \exp[-i\Lambda t]\Phi|\Psi_0\rangle \quad (9)$$

A set of interference amplitudes, with respect to the set of possible vertex-vertex assignments, can be used to check graph matching. For strongly regular graphs, Gamble [24] has given that the graphs can be identified by their class or family with parameters (N, k, λ, μ), each of which graph might be non-isomorphic. Here N is no. of vertices, k is the degree of each vertex, λ is the no. of common neighbors of adjacent vertices, and μ is the no. of common neighbors of non-adjacent vertices. The adjacency matrix of any SRG satisfies the useful relation [26]

$$A^2 = (k-\mu)I + \mu J + (\lambda - \mu)A \quad (10)$$

where I is the identity and J is the matrix of all 1s
We can write

$$A_n = \alpha_n I + \beta_n J + \gamma_n A \quad (11)$$

where α, β and γ are functions only of the family parameters. That is, all SRGs of the same family have the same coefficients.

3.3 Two-particle walks on graphs

Gamble [24] has developed a procedure for each pair of graphs in each family of SRG. It starts with complex evolution matrixU_A. Then the magnitude of each element is considered and all elements are sorted in a list X_A as real values. Such lists for both the matrices are compared.

$$\Delta = \sum_j (X_A[j] - X_B[j]) \quad (12)$$

They have given evolution operator for the interacting Boson case as

$$U = e^{-it\,[-2(I+S)(A\oplus A) + UR]} \quad (13)$$

Expanding as a power series in t, we have

$$U = \sum_{n=0}^{\infty} \frac{(-it)^n}{n!}[-\frac{1}{2}(I+S)(A\oplus A) + UR]^n \quad (14)$$

They have shown that for some sixth order evaluation the algorithm works for up-to N=40 on a high computing cluster.

4. Conclusion

Graph matching is a very fundamental problem in pattern recognition that applies to variety of do-mains. A continuous- time quantum walk has the advantage of being computationally less expensive than discrete-time quantum walk. Single-particle quantum walks are used for the graphs where inexact graph matching is enough goal with some degree of structural errors. Single particle quantum walks as are based on family parameters of SRGs, they cannot distinguish non-isomorphic SRGs. So single-particle quantum walks can be used with different classes of graphs. For exact graph matching and only SRGs, two-particle hardcore boson quantum walk is a powerful method. This method can be applied where insignificant differences can be detected among the objects that resembles to same class of SRGs. Multi-particle quantum random walks with interacting and non-interacting particles may give universal solution.

References

[1] D. A. Spielman, "Faster Isomorphism Testing ofStrongly Regular Graphs", in Proceedings of theTwentyEighth Annual ACM Symposium on Theoryof Computing, STOC-96 (ACM, New York, NY, USA,1996) pp. 576-584.

[2] P.W. Shor, "Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer", SIAM J. Comput., 26 (1997) 1484-1509.

[3] Y. Aharonov, L. Davidovich, and N. Zagury, "Quantum Random Walks", Phys. Rev.A 48, 1687 (1993).

[4] S. Gold, A. Rangarajan, "A graduated assignment algorithm for graph matching", IEEE Trans. Pattern Anal. Mach. Intell. 18 (4), (1996) 377-388.

[5] S. Umeyama, "An eigen decomposition approach toweighted graph matching problems", Pattern Anal.Mach. Intell. 10 (5) (1988) 695-703.

[6] R.C. Wilson, E.R. Hancock, "Structural matching bydiscrete relaxation", IEEE Trans.Pattern Anal. Mach.Intell. 19 (6) (1997) 634-648.

[7] B. Luo, E.R. Hancock, "Structural graph matching using the EM algorithm and singular value decomposition", Pattern Anal. Mach. Intell. 23 (10) (2001) 1120-1136.

[8] Xiao Bai ,Hang Yu ; Hancock, "E.R.Graph matching using spectral embedding and alignment", PatternRecognition, 2004. ICPR 2004. Proceedings of the 17th International Conference on (Volume:3)
[9] Bai Xiao, Edwin R. Hancock, Richard C. Wilson, "Geometric characterization and clustering of graphs using heat kernel embeddings", Image and Vision Computing Volume 28, Issue 6, June 2010, Pages 1003-1021
[10] L. Grover, "A fast quantum mechanical algorithm for database search", in: Proc.28th Annual ACM Symposium on the Theory of Computation, ACM Press, NewYork, NY, 1996, pp. 212-219.
[11] P.W. Shor, "Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer", SIAM J. Comput. 26 (1997) 1484-1509.
[12] J. Kempe, "Quantum random walks –an introductory overview", Contemp. Phys. 44 (4) (2003) 307-327.
[13] E. Farhi, S. Gutmann, "Quantum computation anddecision trees", Phys. Rev. A 58 (1998) 915-928.
[14] V. Gudkov and S. Nussinov, "A Novel ApproachApplied to the Largest Clique Problem", e-print arXiv: cond-mat/0209112.
[15] S.Y. Shiau, R. Joynt, S.N. Coppersmith, "Physicalymotivated dynamical algorithms for the graph isomor-phismproblem", Quantum Inf. Comput. 5 (6) (2005)492-506.
[16] T. Rudolph, "Constructing physically intuitive graph invariants", arXiv:quant-ph/0206068v1 (2002).
[17] J. Kempe, "Quantum random walks hit exponentially faster", in: Proceedings of the 7th International Work-shop on Randomization and Approximation Techniques in Computer Science (RANDOM'03), 2003, pp. 354-369.
[18] D. Aharonov, A. Ambainis, J. Kempe, U. Vazirani, "Quantum walks on graphs", in: STOCâĂŹ01: Proc ACM Theory of Computing, ACM Press, New York, 2001, p 50-59.
[19] A. Childs, E. Farhi, S. Gutmann, "An example of The difference between quantum and classical random walks", Quantum Inf. Process. 1 (2002) 35.
[20] A.M. Childs, R. Cleve, E. Deotto, E. Farhi, S.Gutmann, D.A. Spielman, "Exponential algorithmicspeedup by a quantum walk", in: STOC '03: Proceedings of the 35th ACM Symposium on TOC, New York, NY, USA, ACM Press, New York, 2003,pp. 59-68.
[21] B. Douglas and J. Wang, Journal of Physics A: Mathematical and Theoretical 41, 075303 (2008).
[22] D. Emms, S. Severini, R. C. Wilson, and E. R. Hancock, "Graph matching using the interference of continuous-time quantum walks", Pattern Recog. 42, 1988 (2009).
[23] S. D. Berry and J. B. Wang, Phys. Rev. A 83, 042317(2011).
[24] John King Gamble, Mark Friesen, Dong Zhou, Robert Joynt, and S. N. Coppersmith, "Two-particle quantum walks applied to the graph isomorphism problem", Phys.Rev. A 81, 052313 - Published 13 May 2010
[25] M. A. Jafarizadeha, F. Eghbalifama, S. Namia, "Investigation graph isomorphism problem via entanglement entropy in strongly regular graphs", arXiv-
[26] 1404.4041v1 /quant-ph/2014.C. Godsil and G. Royle, Algebraic Graph The- Ory(Sprin

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