

# A Neural Based Approach for Optimizing Diffusion Wavelet Coefficients

(An application to image denoising)

Kapil Chaudhary<sup>1</sup>, Dr. Narender Kumar<sup>2</sup>

<sup>1</sup>Research Scholar Himalyan University,  
Itangar AP, India

<sup>2</sup>Assistant Professor, Department of Computer Sc. & Engg.,  
H.N.B Garhwal University (A Central University),  
Chauras Campus, Post Office- Kilkleshwar,  
Pin- 249161, Tehri Garhwal, Uttarakhand (INDIA)

## Abstract:

*In this paper ,a special class of wavelets known as Diffusion Wavelets are optimized through their coefficients, the desired process is started by applying the diffusion process to a noisy signal in our case the noisy image. We apply the neural network MLP for the determination of the optimal wavelet basis. Further we fetch the benefit of OWE (over complete wavelet expansion ) over OWT(orthogonal wavelet transform) specially in image diminution problems. The performance of the proposed method is dependent on selection of the wavelet basis.The desired scheme yield fruitful results in image denoising*

**Keywords:** diffusion wavlet ,diffusion packet, OWE, OWT, MLP

## 1. INTRODUCTION

Images are often times corrupted with noise during acquisition, transmission, and retrieval from storage media. Noise disrupts both images and videos [1]. The purpose of the denoising algorithm is to remove such noise. In addition, some fine details in the image may be confused with the noise or vice-versa. Many image-processing algorithms such as pattern recognition need a clean image to work effectively.

Images are affected by different types of noise. Denoising of natural images corrupted by noise using wavelet techniques is very effective because of its ability to capture the energy of a signal in few energy transform values. The wavelet denoising scheme thresholds the wavelet coefficients arising from the wavelet transform. Problem of denoising can be formulated as [2]: Let  $A(i,j)$  be the noise free image and  $B(i,j)$  be the image corrupted with noise  $Z(i,j)$  then

$$B(i,j) = A(i,j) + \sigma * Z(i,j) \quad (1)$$

where  $\sigma$  is the noise variance. In the wavelet domain, the problem can be formulated as

$$Y(i,j) = X(i,j) + W(i,j) \quad (2)$$

Where  $Y(i,j)$  is the noisy wavelet coefficient  $X(i,j)$  is the true coefficient and  $W(i,j)$  is noise. Since the first wavelet soft thresholding approach of Donoho [3], many wavelet-based denoising schemes were reported [4]–[5], [6]–[7], [8], [9], [10]. In all these approaches different wavelets are

applied and accordingly wavelet coefficient are computed in a formal way.

In this paper we select a complete algorithm based on diffusion wavelet-multiscale linear minimum mean square-error estimation (LMMSE) scheme as Kapil et al. [11] and in [12]. In this scheme as a first step a diffusion wavelet is applied on the image which is to be denoised. Although WT well decorrelates signals, but strong intrascale and interscale dependencies between wavelet coefficients may still exist. If a coefficient at a coarser scale has small magnitude, its descendants at finer scales are very likely to be small too. Shapiro [13] exploited this property and developed the well-known embedded zerotree wavelet image compression scheme. In another viewpoint, if a wavelet coefficient generated by true signal has large magnitude at a finer scale, its ascendants at coarser scales will likely be significant as well. But for those coefficients caused by noise, the magnitudes may decay rapidly along the scales. From this observation, it is expected that multiplying the wavelet coefficients at adjacent scales would strengthen the significant structures while diluting noise. Such a property has been exploited for denoising [14]–[15], step estimation [16] and edge detection [17].

The wavelet interscale dependencies have also been represented by Markov models [18], [19]. Some schemes adopted an interscale and intrascale hybrid model to better estimate noisy wavelet coefficients, such as Liu and Moulin [20] and Portilla *et al.* [21]. In [19], each coefficient was modeled as the product of a Gaussian random vector and a hidden multiplier variable to include adjacent scales in the conditioning local neighborhood. The LMMSE denoising schemes in [22] and [23] exploit the wavelet intrascale dependencies. Generally, it is seen that performance of interscale LMMSE scheme is wavelet dependent. A rich library of wavelet bases have been constructed and widely used in signal processing, such as Daubechies' compactly supported orthonormal [24] and biorthogonal wavelets [25]. Our approach is to minimize the wavelet bases searching time, we will achieve this goal by developing a MLP of neural network that smartly chooses the wavelet coefficients. In the recent literature [26] MLP are developed for directly denoising

the images. Although much better results are reported through this technique.

## 2. BREIF REVIEW OF DIFFUSION WAVLETS & PACKETS

Let us consider a semigroup  $\{T^t\}$ , associated to a diffusion process (e.g.  $T = e^{-c\Delta}$ ), here we do not take the Green's operator, since the latter is not available in the applications we are doing, where the space may be a graph and very little geometrical information is available. We utilize the semigroup to induce a multiresolution analysis, interpreting the powers of  $T$  as dilation operators acting on functions, and constructing precise downsampling operators to efficiently represent the multiscale structure. This allows a construction of multiscale scaling functions and wavelets in a very universal setting. The powers of operator  $T$  decreases in rank thus suggesting the compression of the function (and geometric) space upon which each power acts. The scheme consists the following steps: First, apply  $T$  to a space of test functions at the finest scale, compress the range via a local orthonormalization procedure, represent  $T$  in the compressed range and compute  $T^2$  on this range, compress its range and orthonormalize, and so on. At scale  $j$  we obtain a compressed representation of  $T^{2^j}$ , acting on a family of scaling functions spanning the range of  $T^{1+2+2^2+\dots+2^{j-1}}$  basis, and then we apply  $T^{2^j}$ , locally orthonormalize and compress the result, thus getting the next coarser subspace. In [25] the complete construction of diffusion wavelets is given but it only provide the mathematical model for it, Fourier analysis and wavelet analysis. The action of a given diffusion semigroup on the space of functions on the set is analyzed in a multiresolution fashion, where dyadic powers of the diffusion operator correspond to dilations, and projections correspond to down sampling. The localization of the scaling function constructed allows to reinterpret these operations in function space in a geometric fashion.

### Maintaining the Image Denoising parameters

Let us take an image contaminated by noise (without loss of generality, we assume additive noise):

$$I(x) = f(x) + \xi(x)$$

at this juncture our goal is to remove that noise, resulting in minimal damage to the image. Since in most cases the result end user is human, the criterion for denoising fidelity would be the human visual perception of the result, rather than any of known mathematical criteria, such as minimal mean error (MSE) or minimal maximal difference (minimax).

The most adolescent approach would be performing some kind of low-pass filtering on the image, e.g. by convolution of Gaussian. Perona and Malik [26] claim that low-pass filtering by Gaussian kernel convolution can be equivalently formulated as forward diffusion

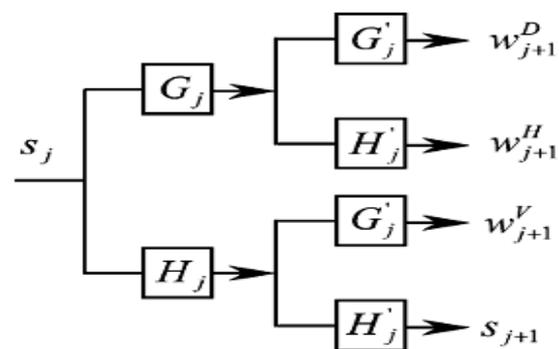
$$\frac{\partial I(x; t)}{\partial t} = c \nabla^2 I(x; t) \quad c = \text{const}$$

where  $t$  is the time parameter proportional to the Gaussian standard deviation and  $c$  is the diffusion coefficient. The disadvantage of low-pass filtering or homogenous adaptive diffusion in image enhancement applications is the fact that Gaussian blurring does not respect the natural edges of the image. If we know that a class of functions is well compressed by wavelet packets, then by thresholding the coefficients, we expect to be able to denoise functions from the class (assuming, of course, that the 'noise' is not well compressed by wavelet packets). Efficient and asymptotically optimal denoising algorithms for denoising have been studied by Donoho and Johnstone [27].

## 3. DIFFUSION WAVLETS OPTIMIZATION PROCEDURE

As we analyze in the previous section that OWE achieves better results than traditional OWT specially in the denoising environment. Our work focus only OWE. Orthogonal wavelet transform (OWT) is translation variant due to the downsampling. This will cause some visual artifacts (Gibbs phenomena) in threshold-based denoising [27][28].

It has been pragmatic that the OWE (undecimated WT or translation-invariant WT in other names) achieves better results in noise reduction and artifacts suppression [29], [30], [31], [32]. The denoising scheme presented in this paper adopts OWE, whose one stage two-dimensional (2-D) decomposition structure is shown in Fig. 1, just as an illustration how it works analytically.



**Figure. 1** Decomposition of the 2-D OWE.  $w_{jH}$   $w_{jV}$  and  $w_{jD}$  are the wavelet coefficients at horizontal, vertical and diagonal directions.

It is eminent from [33][34] that the wavelet-represented images are similar across scales, especially among the adjacent scales. In wavelet domain, the noise level decrease rapidly along scales, while signal structures are strengthened with scale increasing. Consequently we use coarser scale information to improve finer scale estimation. Assume the input image is decomposed into scales. generally it can be made that,  $j$  scale is strongly correlated with scale.  $j+1$ , but its correlations with  $j+2$  and so on will decrease rapidly. These scales would not provide much additional information to improve the estimation of scale  $j$ .

Now we turn our direction to model the wavelet to neural network MLP modelling so that the wavelet coefficients are optimized in terms of directly their computation. We are using MLP because convolutional neural network have different architecture from MLP which definitely complicate learning process. The learning process directly depends on features count, which for image is its height multiplied by width in pixels. An example model of MLP is shown in Fig 2

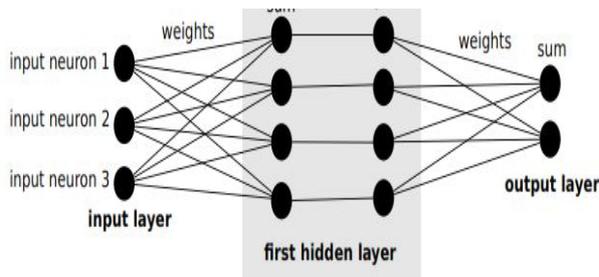


Figure 2. A graphical representation of a (3,4,2)-MLP.

Generally, the CNN consist of multiple convolution and sub sampling layers, where convolution and subsampling layers change each other. Each of the layers includes several features maps, which are connected to previous layer maps through set of small receptive fields, as shown in Fig 3.

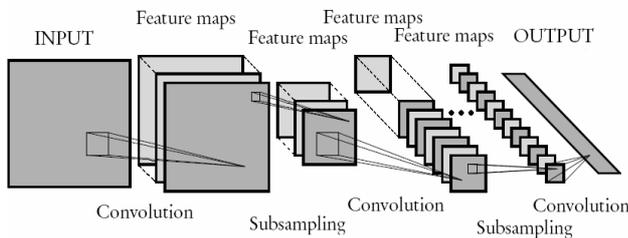


Figure.3 CNN architecture

Every subsampling layer is followed by convolution layer. In this case connections between layers defines interconnection matrix. This process repeats until features maps become too small, e.g. 1x1. After this feature maps is followed by full interconnected MLP which output is classifiers vector. The idea of performing subsampling layer is shown on fig. 4.

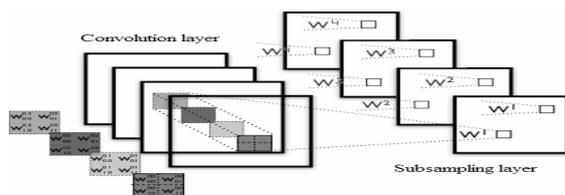


Figure 4 Subsampling layer formation

Assume there is input data which is represented by  $X$  matrix and  $W^{nm}$  is kernel matrix, where  $m$  is kernel number of  $n$ -th convolution layer (1). The result of convolution operation is set of features maps  $M^{nm}$ , where

$m$  feature map number of  $n$ -th convolution layer as given in below equation 3

$$M^{lm}[k,l] = \sum_{l'=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} X[l-l',k-k'] \cdot W^{lm}[l',k'], \quad (3)$$

Where  $k,l$  is feature for  $m$ th map. Therefore, after applying transform only part of coefficients can be passed to CNN. This may reduce not only features number but as the result simplify CNN architecture. For input data features submatrices  $A, D, D_v, D_h, D_d$  may be used. But as it will be realize in experiments the most fitting results, with the same misclassification rate as original, will give approximation matrix  $A$ .

#### 4. IMAGE DENOISING SCHEME

The image denoising procedure will be proceed as shown in fig 5



Figure 5 Image denoising procedure

(i) The image is converted to small image patches through simple pre-processing in MATLAB. The reason is simple because it is easy to process image patches in step iv and v.

(ii) We apply Orthogonal Wavelet Expansion as illustrated in previous section with wavelet applied as a diffusion process.

(iii) Find out the vector as from eq.3.

(iv) Next step is to find out optimally wavlet basis, for that we use the neural network MLP approach as illustrated.

(v) Accordingly, the wavlet coefficient are adjusted.

Step 4&5 will correspond to likely constructing the sparse dictionaries for diffusion wavlets.

(vi) Apply IWT (Inverse Wavlet Transform).

(vii) Denoised Patch is the output.

A comparison of the approach to other algorithms is shown in Table 1. In terms of a Mean square error table. Where  $\sigma$  is the standard deviation. A smaller mean square error indicates that the estimate is closer to the original image. The numbers have to be compared on each row. The square of the number on the left hand column gives the real variance of the noise. By comparing this square to the values on the same row, it is quickly checked that all studied algorithms indeed perform some denoising. In

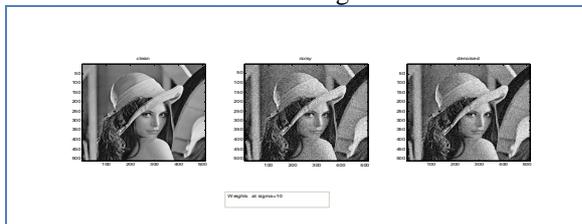
general, the comparison performance corroborates the previously mentioned quality criteria.

**Table 1.** Comparison of MSE for various image Denoising Algorithm

$\sigma$	MSE Table		
	Image	Algorithm	MSE
8	Lena	GF	57
10	Lena	AF	104
25	Peepers	TV	181
35	House	YNF	382
45	Cameraman	Proposed	72

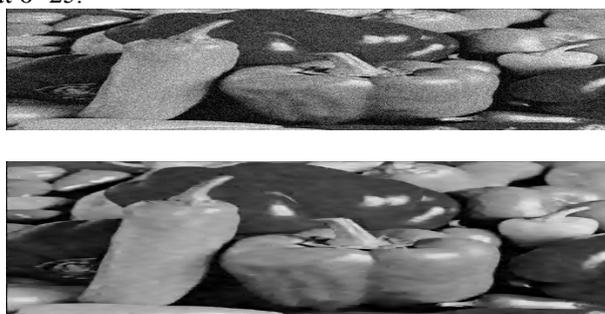
### 5. RESULTS & CONCLUSION

Test images basically, three famous original images (256x256 grayscale) as shown in fig 4 are taken for experimental purpose from URL Index of /~phao/CIP/Images [www.eecs.qmul.ac.uk](http://www.eecs.qmul.ac.uk). The (a) is the famous Lena image, (b) is the Peepers, (c) House, (d) Cameraman. At first glance a random noise is added to the image then apply the procedure as described in fig 3. Undoubtedly, the denoised version of the respective images is dependent on the value of sigma (Noise Variance). Let us take  $\sigma=10$ , at this small value our results are shown in fig 6.



**Figure 6** Rightmost denoised image at  $\sigma=10$

For simulation purpose different .mat files of neural weights, for instance in fig 6  $\sigma=10$  is occupied. It is evident from the experiment that at lower values of  $\sigma$  the proposed scheme is not successfully fighting fit. Hence, the training images is to be increased so that more choice of weights will come up. The next fig 7 will show this piece of evidence. It comprises both noisy and clean image at  $\sigma=25$ .



**Figure 7:** Noisy image (a) top (b) just below clean image at  $\sigma=25$

As the  $\sigma$  is increased better results can be obtained. Finally, the conclusion is drawn that paper presented an optimized Diffusion Wavelet based novel image denoising approach for digital camera imaging applications. To fully exploit the spatial and spectral correlations of the images during the denoising process, the training samples from different  $\sigma$  were localized by using a supporting MLP structure of neural network. With OWE the wavelet coefficients at the same spatial locations at two adjacent scales are represented as a vector and then LMMSE is applied to the vector. The wavelet interscale dependencies are thus subjugated to improve the signal estimation. The overall performance of the scheme is dependent on the neural weight adjustment guiding principle. The vector find out though LMMSE is further optimized using MLP. MLP help to artificially find out the optimal solution to the problem of finding out the optimal wavelet bases. The proposed scheme also preserves very well the fine structures in the image, which are often smoothed by other denoising schemes.

### References

- [1]. R.E. Woods, R.C. Gonzalez, "Digital Image Processing", Pearson Prentice Hall, 3<sup>rd</sup> edition, 2009.
- [2]. G.Y. Chen, T. D. Bui and A. Krzyzak, "Image denoising using neighbouring wavelet coefficients", IEEE, pp. II (917-920), ICASSP, 2004.
- [3]. L. Donoho, "De-noising by soft thresholding," IEEE Trans. Inform. Theory, vol. 41, no. 5, pp. 613-627, May 1995..
- [4]. S. G. Chang, B. Yu, and M. Vetterli, "Adaptive wavelet thresholding for image denoising and compression," IEEE Trans. Image Process., vol. 9, no. 9, pp. 1532-1546, Sep. 2000. I.S. Jacobs and C.P. Bean, "Fine particles, thin films and exchange anisotropy," in Magnetism, vol. III, G.T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271-350.
- [5]. P. Bao and L. Zhang, "Noise reduction for magnetic resonance images via adaptive multiscale products thresholding," IEEE Trans. Medical Imaging, vol. 22, no. 9, pp. 1089-1099, Sep. 2003. "shrinkage," Biometrika, vol. 81, pp. 425-455, 1994..
- [6]. Chambolle, R. A. Devore, N. Y. Lee, and B. J. Lucier, "Nonlinear wavelet image processing: variational problems, compression, and noise removal through wavelet shrinkage," IEEE Trans. Image Process., vol. 7, no. 7, pp. 319-335, Jul. 1998
- [7]. Li and M. Orchard, "Spatially adaptive image denoising under overcomplete expansion," in Int. Conf. Image Process., Vancouver, Canada, Sep. 2000, pp. 300-303.
- [8]. M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989.

- [9]. "Image denoising using local contextual hidden Markov model in the wavelet domain," *IEEE Signal Process. Lett.*, vol. 8, no. 5, pp.125–128, May 2001.
- [10]. "Image denoising based on scale-space mixture modeling for wavelet coefficients," in *Proc. ICIP'99*, Kobe, Japan, Oct. 1999, pp. I.386–I.390.
- [11]. Kapil Chaudhary, Dr. Narendra Kumar, Dr. Vipin Kumar Saini, "A New Approach to Image Denoising based on Diffusion-MLP-LMMSE Scheme". *International Journal of Software and Web Sciences, IASIR*, ISSN(print):2279-0063, ISSN(online) 2279-0071 ([www.iasir.net](http://www.iasir.net))
- [12]. J. Portilla, V. Strela, M. J. Wainwright, and E. P. Simoncelli, "Adaptive wiener denoising using a gaussian scale mixture model in the wavelet domain," in *Proc. 8th Int. Conf. Image Processing*, vol. 2, Thessaloniki, Greece, Oct. 2001, pp. 37–40.
- [13]. J. M. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients," *IEEE Trans. Signal Process.*, vol. 41, no. 12, pp. 3445–3462, Dec. 1993.
- [14]. Y. Xu, J. B. Weaver, D. M. Healy, and J. Lu, "Wavelet transform domain filters: a spatially selective noise filtration technique," *IEEE Trans. Image Process.*, vol. 3, no. 11, pp. 747–758, Nov. 1994.
- [15]. P. Bao and L. Zhang, "Noise reduction for magnetic resonance images via adaptive multiscale products thresholding," *IEEE Trans. Medical Imaging*, vol. 22, no. 9, pp. 1089–1099, Sep. 2003.
- [16]. M. Sadler and A. Swami, "Analysis of multiscale products for step detection and estimation," *IEEE Trans. Inform. Theory*, vol. 45, no. 4, pp. 1043–1051, April 1999.
- [17]. L. Zhang and P. Bao, "Edge detection by scale multiplication in wavelet domain," *Pattern Recognit. Lett.*, vol. 23, pp. 1771–1784, 2002.
- [18]. M. Crouse, R. Nowak, and R. Baraniuk, "Wavelet-based statistical signal processing using hidden Markov models," *IEEE Trans. Signal Process.*, vol. 42, no. 4, pp. 886–902, Apr. 1998.
- [19]. G. Fan and X. G. Xia, "Improved hidden Markov models in the wavelet domain," *IEEE Trans. Signal Process.*, vol. 49, no. 1, pp. 115–120, Jan. 2001.
- [20]. "Image denoising based on scale-space mixture modeling for wavelet coefficients," in *Proc. ICIP'99*, Kobe, Japan, Oct. 1999, pp. I.386–I.390.
- [21]. J. Portilla, V. Strela, M. J. Wainwright, and E. P. Simoncelli, "Adaptive wiener denoising using a gaussian scale mixture model in the wavelet domain," in *Proc. 8th Int. Conf. Image Processing*, vol. 2, Thessaloniki, Greece, Oct. 2001, pp. 37–40.
- [22]. M. K. Mihçak, I. Kozintsev, K. Ramchandran, and P. Moulin, "Low complexity image denoising based on statistical modeling of wavelet coefficients," *IEEE Signal Process. Lett.*, vol. 6, no. 12, pp. 300–303, Dec. 1999.
- [23]. Li and M. Orchard, "Spatially adaptive image denoising under overcomplete expansion," in *Int. Conf. Image Process.*, Vancouver, Canada, Sep. 2000, pp. 300–303.
- [24]. Daubechies, "Orthonormal bases of compactly supported wavelets," *Comm. Pure Appl. Math.*, vol. 41, pp. 909–996, 1988.
- [25]. A. Cohen, I. Daubechies, and J. C. Feauveau, "Biorthogonal bases of compactly supported wavelets," *Comm. Pure Appl. Math.*, vol. 45, pp. 485–560, 1992.
- [26]. Ronald R. Coifman, Mauro Maggioni, Program in Applied Mathematics, Department of Mathematics Yale University U.S.A.
- [27]. P. Perona, J. Malik, Scale-space and edge detection using anisotropic diffusion, *Transactions on Pattern Analysis and Machine Intelligence*, vol. 12 No. 7 July 1990
- [28]. L. Donoho and I. M. Johnstone, "Adapting to unknown smoothness via wavelet shrinkage," *J. Amer. Stat. Assoc.*, vol. 90, pp. 1200–1224, Dec. 1995.
- [29]. R. R. Coifman and D. L. Donoho, "Translation-invariant de-noising," in *Wavelet and Statistics*, A. Antoniadis and G. Oppenheim, Eds. Berlin, Germany: Springer-Verlag, 1995.
- [30]. S. G. Chang, B. Yu, and M. Vetterli, "Adaptive wavelet thresholding for image denoising and compression," *IEEE Trans. Image Process.*, vol. 9, no. 9, pp. 1532–1546, Sep. 2000.
- [31]. R. R. Coifman and D. L. Donoho, "Translation-invariant de-noising," in *Wavelet and Statistics*, A. Antoniadis and G. Oppenheim, Eds. Berlin, Germany: Springer-Verlag, 1995.
- [32]. Q. Pan, L. Zhang, G. Dai, and H. Zhang, "Two denoising methods by wavelet transform," *IEEE Trans. Signal Process.*, vol. 47, no. 12, pp. 3401–3406, Dec. 1999.
- [33]. X. Li and M. Orchard, "Spatially adaptive image denoising under overcomplete expansion," in *Int. Conf. Image Process.*, Vancouver, Canada, Sep. 2000, pp. 300–303.
- [34]. M. Filippone et al. A survey of kernel and spectral methods for clustering. *Pattern Recognition*, 41:176{190, January 2008