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Blind Spot Effect During Genetic Programming Based Inference of Dynamical Model Equations for Chaotic Systems

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Abstract

A study based on Genetic Programming (GP) framework is carried out for the inference of model equations for some well known dynamical systems that give rise to chaotic time series. The systems studied are Lorenz and Rossler systems (involving nonlinear coupled Ordinary Differential Equations (ODEs)) and Henon attractor (having coupled nonlinear map equations). The solutions of these systems in specific parameter regimes lead to chaotic series that are too sensitive to initial conditions. The inherent nonlinearity and chaotic nature of the time series makes the inference of system equations very challenging and computationally a difficult problem. Additionally, a plausible reason for the difficulty is attributed to an interesting observation regarding an effect, we call as a *Blind Spot Effect*. Often during the exploration of the standard GP search space, symbolic chromosome structures padded with spurious terms settle down with *higher* fitness value creating local maxima. Consequently, the GP search method finds it difficult to shed away the spurious terms leading either to a failure, or at most a very slow convergence in getting around such local maxima. A possible solution to this problem is effected in the present work by combining the standard GP search method with routines that directly help get around the blind spot effect. The resulting enhanced GP method is found to be an empirically successful method to deduce the ODE equations with very good accuracy for the systems studied.

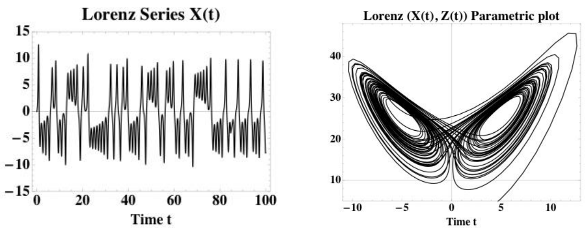
Index Terms: Genetic Programming, Blind Spot Effect, Inference of System Equations, Symbolic Computation.

1. INTRODUCTION

A vast multitude of physical systems in diverse areas of science, including engineering, physics, biology, economics etc are known to successfully capture their dynamical description quite succinctly in terms of a mathematical model based on coupled ordinary/ partial differential equations (ODEs/ PDEs). Given the system equations and the initial conditions, an apparent assumption in the determinism for the space/ time profile is well known to be shattered for the systems that show chaos. Chaos is essentially the occurrence of such unpredictable behavior arising out of deterministic systems. Consider the Lorenz system (Edward Lorenz [1], Lorenz [2], Sparrow [3], Pierre et al. [4]) comprising of three coupled nonlinear ODE equations as shown in Equation (1),

$$\frac{dX}{dt} = \sigma(X(t) - Y(t))$$
$$\frac{dY}{dt} = rX(t) - Y(t) - X(t)Z(t) \qquad (1)$$
$$\frac{dZ}{dt} = X(t)Y(t) - bZ(t)$$

Here (σ, r, b) are system parameters, and a specific choice for parameter values (-3, 26.5, 1) is known to generate solution in a chaotic regime. Using the initial conditions as X(0) = Z(0) = 0.0 and Y(0)=1.0, the system equations are solved to obtain the time profiles for X(t), Y(t) and Z(t) series, and Fig. 1 (a) shows one of these plots for X(t). Interestingly, the aspect of chaos is revealed through a strange attractor referred to in the literature as butterfly effect (Fig. 1 (b)), through a parametric plot for the phase space trajectory for X(t) versus Z(t).



(a) (b)

Figure 1 (a) Time profile for X(t), and (b) parametric plot X(t) versus Z(t) for Lorenz system (with system parameters σ=-3, r=26.5 and b=1), showing chaotic time series and strange attractor respectively.

The aim in the present work is to use the numerical data, such as the one represented in Fig. 1 (a) and use GP method to infer the system equations. In this regards it is well known that the inverse problem of re-engineering the system equations from its known data (in numeric/ symbolic form) is quite a complex and challenging task. The standard GP approach attempts at finding a symbolic solution through a fitness driven approach to minimize the cumulative error in fitting the a given set of numerical data. This method, also known as symbolic regression technique, is successful for a wide range of applications in the area of solving ODEs/ PDEs (Tsoulos and Lagaris [5]) as well as its inverse problem, i.e. inference of differential equations from known data (Koza [6], Gray et al [7] and Cao et al [8]). It is also known that due to the stochastic approach deployed by GP, it generates a very large search space and often misses getting the maximum possible fitness required to get an exact inference of equations. This has indeed been one of the reasons for augmenting the standard GP framework with additional tools to speed up the search. GP method has been applied earlier for the prediction of chaotic time series by Mulloy [9]. Zhang et al [10] used a Genetic Programming Modeling (GPM) algorithm for modeling chaotic time series for Logistic Map in which GP is used for searching structures in function space and a Particle Swarm Optimization (PSO) is used for nonlinear parameter estimation. Inference of ODEs by Multi Expression Programming (MEP) for Gene Regulatory Networks has been carried out by Bin Yang et al [11]. Earlier Sakamoto and Iba [12] have also applied Genetic Programming for identifying Gene Regulatory Network as Differential Equations.

While delving upon the inverse problem of inference of model equations from its known data, an important result of the present work is the observation of a Blind Spot Effect related to bloating of the chromosomes by spurious terms that is found to slow down the standard fitness driven GP search procedure. In the present work the standard GP program is adequately enhanced with routines to address this problem effectively.

The paper is organized as follows. In Section 2 the GP framework is elaborated with reference to inference of dynamical system equations involving ODE equations. The *Blind Spot effect* is described in a schematic manner in Section 3 as a hurdle for the standard GP search. Section 4 describes the enhanced GP approach to resolve the blind spot effect. Section 5 describes results obtained by GP experiments, and by considering three chaotic systems, namely Lorenz system, Rossler system and Henon map in Sections 5.1, 5.2 and 5.3 respectively. Through these GP experiments the occurrence of Blind Spot effect is seen to arise in a practical scenario, and it is interesting to note that the enhanced GP search framework deployed in the present work overcomes the hurdle and successfully generates exact solutions for the three chaotic systems considered. Finally the conclusion and outlook are presented in Section 6.

2. GP OPTIMIZATION TECHNIQUE FOR INFERENCE OF ODE EQUATIONS

We follow the standard GP optimization approach as pioneered by John R. Koza [6]. In order to infer the ODE equations from known data, the general form for the ODE equations for Lorenz system involving 3 independent variables (X, Y, Z) is shown in Equation (2),

$$\frac{dX}{dt} = f_1(X,Y,Z,t) \qquad (2)$$
$$\frac{dY}{dt} = f_2(X,Y,Z,t)$$
$$\frac{dZ}{dt} = f_3(X,Y,Z,t)$$

Here the order of ODE equations is assumed to be 1; however it ought to be pointed out that the method is equally applicable for higher order equations as well. Thus the GP search method is used as a means to grow the symbolic structure on the right hand sides of Equation (2) by minimizing the cumulative error E in Equation (3), where we have considered the dX/dt equation in Equation (2),

$$E = \sum_{i=1}^{i=N} \left(\frac{dX}{dt}^{given}(i) - \frac{dX}{dt}^{calc}(i) \right) \qquad (3)$$

Here N is the number of data points considered, $dX/dt^{given}(i)$ are numerically supplied derivatives, and $dX/dt^{calc}(i)$ are the derivatives calculated using the evolved symbolic expression on the right hand side of Equation (2). It may be noted that the numerical derivatives required to be calculated for the given numerical data for (X(t), Y(t), Z(t)) are calculated using a standard 5-point central difference formula [13]. A fitness measure is scaled in the range [0, 1] such that the maximum fitness equals 1.0 when the cumulative error is 0. Similar calculations are also separately carried out for the inference of other two equations for dY/dt and dZ/dt in Equation (2).

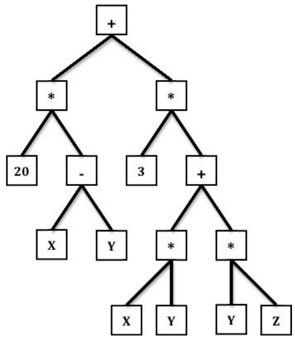


Figure 2 Symbolic binary tree representation for the algebraic expression 20 (X-Y) + 3 (X Y + Y Z).

As an example, Fig. 2 corresponds to a tree representation for a possible symbolic expression that may evolve during GP optimization for the right hand side of the first equation in Equation (2) leading to the ODE equation dX/dt = 20(X-Y) + 3(XY+YZ). The parameters of the standard GP framework used in various GP experiments are shown in Table 1.

Table 1: GP parameters used for various GP experiments

Parameter	Values
Terminal set	Numbers and x,y,dx/dt, dy/dt, dz/dt, where 0.5 is used as a cutoff for either selecting a number of any other member from the terminal set and the random number being selected is in the range [0, 1].
Function set	+, -, *, \
Chromosome template	((A □ B) □ (C □ D)) where □ denotes an operator from function set and A,B,C,D denote an element from terminal set.

Elitism	Best 5% chromosome are copied to next generation
Selection policy	Tournament selection
Crossover	One-point crossover
Probabilities	0.9 for crossover and 0.1 for mutation

3. BLIND SPOT EFFECT AS A HURDLE FOR GP SEARCH

A standard GP search is prone to many calculational challenges, including bloating of chromosomes, repetitive occurrence (i.e. flooding) of a chromosome within the pool of chromosomes, probable singularity problem, inherent nonlinearity in the system equations, failure to achieve solutions having maximum fitness=1.0 exactly due to trapping in local maxima etc.

In particular, the latter difficulty is found to occur more frequently during inference of ODE equations considered here for chaotic time series data, and this is highlighted by noting an effect, we call as a Blind Spot Effect. The effect arises when the fitness driven optimization procedure generates a bloated chromosome, whose structure is very similar to the exact solution except for the fact that it contains few spurious terms as against an additional missing term. Therefore the spurious terms need to be shed away and replaced by the required missing term by the GP method; however if the bloated chromosome has a local maxima such that the removal of spurious terms alone (i.e. without the replacement of the missing term) leads to a *decrease* in the fitness value, then the blind spot effect ensues. This is because the GP procedure would consequently find it difficult to get rid of the spurious terms. We call this as a blind spot effect because the desired exact solution happens to be so close to discover as far as chromosome structure is concerned, and on the other hand it remains unnoticed by the GP procedure based on the stringent fitness driven approach. Though the genetic operators are in principle capable of taking care of this effect, the desired transformation through a purely stochastic crossover/ mutation operators becomes highly improbable. In other words, GP method is required to carry out a two-step process, namely

1) removal of spurious terms, and
2) addition of required missing term

which is a complex transformation to achieve as far as the standard GP optimization is concerned. The net effect is a considerable slowing down in the GP iterative procedure (and often a failure) for marching towards the expected result of achieving the maximum fitness=1.0. The effect is illustrated by a schematic example in Table 2.

Table 2: A typical scenario for the occurrence of Blind Spot effect.

Fitness	**Chromosome**
0.8274	$\mathbf{t_1}^{\mathbf{Good}} + t_2^{Spurious}$
0.5519	$\mathbf{t_1}^{\mathbf{Good}}$
1.0	$\mathbf{t_1}^{\mathbf{Good}} + \mathbf{t_3}^{\mathbf{Good}}$

Here the bloated chromosome ($\mathbf{t_1}^{\mathbf{Good}} + t_2^{Spurious}$) having a fitness value of 0.8274 has 2 terms; the first term being good (shown in bold) and the second term being spurious. The throwing away of the spurious term would be discouraged by GP search procedure, as it would lead to a substantial decrease in the fitness value to 0.5519. If the spurious term is removed and the remaining good term $\mathbf{t_3}^{\mathbf{Good}}$ is added, then the resulting chromosome ($\mathbf{t_1}^{\mathbf{Good}} + \mathbf{t_3}^{\mathbf{Good}}$) would acquire the desired maximum fitness value equal to 1.0, implying that it corresponds to the desired exact solution. It may be noted that this scenario is in fact an actual result observed during inference of ODE equation for dZ/dt for the Lorenz system, and the detailed results are presented later in Section 5.1.

The above example captures the essence of the blind spot effect, and the name is given after the well known blind spot (also called scotoma) effect in our human eyes corresponding to a blockage of visual field due to lack of light-detecting photoreceptor cells. As is well known, due to the blind spot, a part of the visual scene in front of the eyes remain undetected, even though it would be well within the visual field. Similar effect arises in many other areas also. For example in the game of Chess, typical situations may arise, whereupon a well planned sacrificial play would lead to a winning game soon after. In such dramatic situations a player intentionally allows for him to undergo a loss in material (e.g. a rook/ queen sacrifice) leading to a temporary loss of material; however the player may gain a much bigger advantage in tactical/ positional play or even force a checkmate during the subsequent play.

4. THE ENHANCED GP METHOD

Based on our previous work on Genetic Programming for modeling time series data of real systems [14] and the Sniffer technique for efficient deduction of model equations [15], a Mathematica code is indigenously developed for the inference of model equations given the time series data in its numerical/ symbolic form. The code is augmented by appropriate routines for adequately dealing with and getting around the blind spot effect, as will be described soon.

4.1 HEURISTIC MODIFICATIONS IN CHROMOSOMES BASED ON TWO-STEP CORRECTION

As illustrated in Section 3, getting around the blind spot effect requires a two step correction procedure, i.e. removal of spurious terms in the bloated chromosomes and its replacement by the appropriate missing terms, as described below,

1) Step 1: Removal of spurious term(s)

Identify frequently occurring bloated chromosomes containing a common symbolic expression (shown in bold in Table 3). Refine bloated chromosome by preserving the common part and throwing away remaining terms. As is described earlier, this removal leads to a cleaner chromosome resulting in a *decrease* in fitness value, and yet it is preserved for further analysis.

2) Step 2: Add a randomly generated corrective term

A stochastically generated symbolic expression (as per the chromosome template shown in Table 1) is then added to the cleaner chromosome obtained in Step 1 to check if the fitness value increases. If an appropriate term is added during this process, it is likely that the maximum possible fitness value 1.0 can be achieved by the heuristically guided GP method.

Such heuristic tickling of chromosome variations is performed not at each iteration, but after the elapse of a predefined number of iterations (kept typically at 5 iterations in the present experimentation) to allow the GP procedure to relax on the fitness terrain.

Thus the enhanced GP program includes the following two components,

1) a standard GP method as described in Section 2
2) routines to identify, analyze and possibly get around the bloated chromosomes generated due to blind spot effect.

5. GP EXPERIMENTATION

GP experiments have been carried using enhanced GP method for the 3 chaotic systems, namely Lorenz, Rossler and Henon Map, and the results are discussed below.

5.1 LORENZ SYSTEM

First we consider the Lorenz system, where the numerical data for dX/dt^{given}, dY/dt^{given} and dZ/dt^{given} values are used to carry out GP optimization separately for the three coupled equations shown in Equation (2).

Table 3: Chromosomes and their fitness values generated during GP experiments for Lorenz equations showing the occurrence of blind spot effect.

Equation dZ/dt = XY - Z			
Fitness	Chromosome	Fitness	Chromosome
0.8274	**XY** -9.65	0.6374	**XY** +Y +1.4/Z -3.6
0.8101	**XY** -X/t -8.9	0.6236	**XY** -1/Y -2.8
0.7861	**XY** -X -9.1	0.6082	**XY** -1.61
0.7764	**XY** -0.18t	0.5970	**XY** -Z/t
0.7202	**XY** -Y -7.1	0.5722	**XY** -X +Y -1
0.6663	**XY** $-X^2$		
0.6399	**XY** $-X -X^2$ -2.3	0.5519	**XY**

As an illustration, Table 3 lists some of the frequently occurring bloated chromosomes along with their fitness values observed during the inference of dZ/dt equation. For example the chromosomes **XY**-0.18t and **XY**-X^2 have fitness values 0.7764 and 0.6663 respectively, and if their spurious terms are removed, then the resulting chromosome **XY** has the substantially lower fitness value 0.5519. Here onwards, the fitness values and numerical coefficients within chromosomes are shown with 4 and 2 significant digits after decimal point respectively.

It is noted that the enhanced GP program is successful in finding the three ODE equations accurate up to six significant digits and with fitness value 1.0, having the same form as in Equation (1), where the system parameters found by GP aare $(\sigma, r, b)_{GP}$ = (-2.999999, 26.499999, 0.999999). It is interesting to compare the plots of the time series of Lorenz system, for X(t), as generated by

1) the original Lorenz equations (Equation (2)), and
2) from the inferred equations obtained by the enhanced GP method.

In Fig. 3 these two plots for X(t) have been shown (Fig. 3 (a) and Fig. 3 (b) for $X^{Given}(t)$ and $X^{GP}(t)$ respectively) and also shown is the plot for the difference between these two series (Fig. 3 (c)). It is seen that the solution of inferred Lorenz system reproduces the X(t) time series well up to around t=20 time steps, after which the cumulative difference becomes large enough for the chaotic system to grow it further substantially. This is not unexpected and can be understood by noting that a chaotic system is too sensitive to the initial conditions.

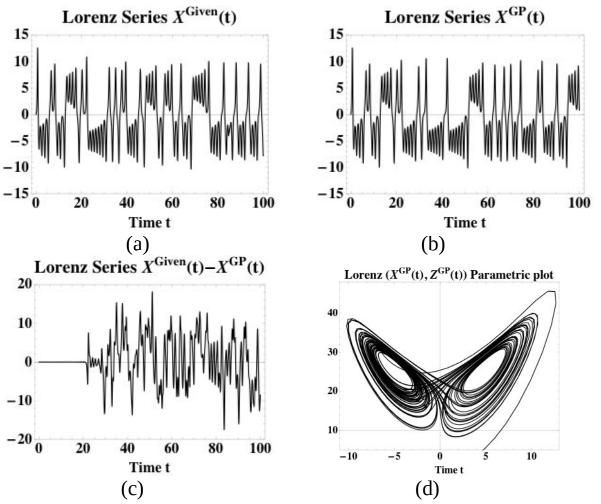


Figure 3 The plots for Lorenz solutions: (a) original Lorenz series $X^{Given}(t)$, (b) Lorenz series $X^{GP}(t)$, (c) the difference in two series $X^{Given}(t)$-$X^{GP}(t)$ and (d) parametric plot $X^{GP}(t)$ versus $Z^{GP}(t)$.

Also, it is noted that the symbolic equations inferred by GP search match very closely with the exact equations of Equation (2) with an accuracy of six decimal digits, and this small departure in the system parameters, namely for (σ, r, b) leads to the difference in the two series as shown in Fig. 3 (c). Interestingly, the parametric plot obtained by GP inferred solution is well reproduced, as can be seen by comparing Fig. 3 (d) with Fig. 1 (b).

5.2 ROSSLER SYSTEM

GP experiment is further carried out in a similar way for the Rossler system made up of three coupled nonlinear ODE equations as shown in Equation (4),

$$\frac{dX}{dt} = -(Y(t) + Z(t))$$
$$\frac{dY}{dt} = X(t) + AY(t) \qquad (4)$$
$$\frac{dZ}{dt} = B + X(t)Z(t) - CZ(t)$$

The set of ODE equations are first solved using the system parameters A=B=0.2 and C=5.7, and initial conditions X(0)=Z(0)=0.1, Y(0)=1.0.

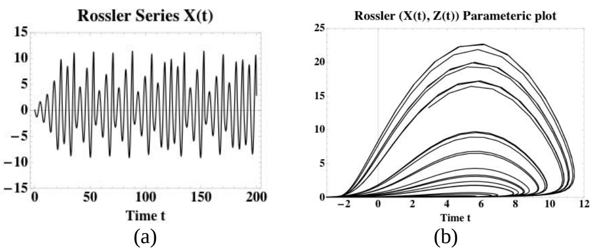


Figure 4 (a) The time profile for X(t) series and (b) the parametric plot (X(t) versus Z(t)) for the Rossler system.

The time profile for X(t) and a parametric plot for X(t) versus Z(t) are shown in Fig. 4 (a) and Fig. 4 (b) respectively, where a strange attractor is noticeable in Fig. 4 (b).

Table 4: Chromosomes and their fitness values generated during GP experiments for Rossler equations showing the occurrence of blind spot effect.

Equation dX/dt = -Y – Z		**Equation dZ/dt = -5.7Z +XZ +0.2**	
Fitnes s	Chromosome	Fitness	Chromosome
0.7905	**-Y** – XZ/t	0.9995	**-5.7Z + XZ** +0.16Y²/t
0.7804	**-Y** -0.16X	0.9995	**-5.7Z + XZ** +Z/t
0.7688	**-Y** -0.3	0.9994	**-5.7Z + XZ** +0.18XZ/t
0.7651	-Y -0.34X²/t	0.9994	**-5.7Z + XZ** +XZ/t²
		0.9994	**-5.7Z + XZ** +0.12Z/(tY)
0.7569	**-Y**	0.9994	**-5.7Z + XZ** +0.36YZ/t
		0.9994	**-5.7Z + XZ** +0.02Z/t
0.3340	**-Z** +1	0.9994	**-5.7Z + XZ** +XZ/t²
0.3290	**-Z** –X/t -1		
0.3197	**-Z** +X/t	0.9991	**-5.7Z + XZ**
0.3164	**-Z** -0.2XZ/t		
0.3161	**-Z**		

Using the time series data generated by solving the system equations for Rossler system, the inverse calculation for the inference of the symbolic equations is carried out using the enhanced GP method. Table 4 lists some of the bloated chromosomes corresponding to dX/dt and dZ/dt equations in Equation (4) that give rise to blind spot effect for the Rossler system. Interestingly, for the dX/dt equation, the effect is observed for two cases corresponding to the terms for -Y and -Z, though due to the higher fitness value for -Y, namely 0.7569, the occurrence of the effect is observed more frequently for the chromosome -Y. For the dZ/dt equation, the effect is even more pronounced due to the relatively higher value of fitness=0.9991 for the chromosome **-5.7Z + XZ**. Interestingly, for the dY/dt equation (not shown in Table 4), the effect is prominent for the chromosome X due to relatively high fitness value of 0.9620, but not for the chromosome Y, due to its low fitness value 0.1124.
The enhanced GP method finds the three coupled ODE equations having the best possible fitness value 1.0 having 6 significant digits accuracy, and the inferred equations have same symbolic form as in Equation (4), with system parameters as $(A,B)_{GP}$=0.199991 and C_{GP}=5.699991.

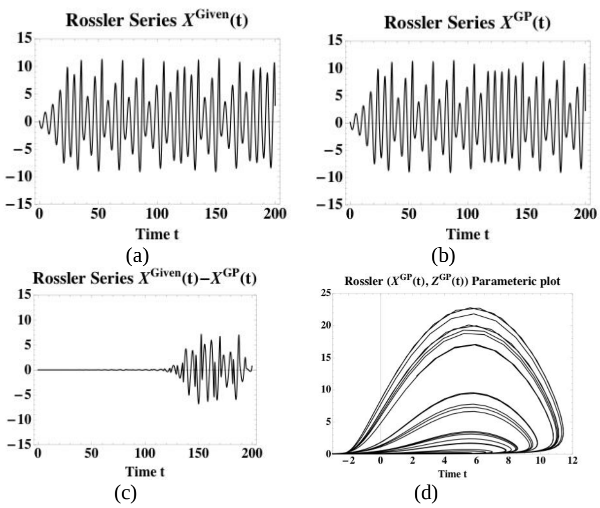


Figure 5 The plots for Rossler system: (a) original Rossler series $X^{Given}(t)$, (b) GP inferred Rossler series $X^{GP}(t)$, (c) the difference in two series $X^{Given}(t)-X^{GP}(t)$ and (d) parametric plot $X^{GP}(t)$ versus $Z^{GP}(t)$.

In Fig. 5 the time profiles and parametric plots for the solutions as given by the GP inferred equations for Rossler system are shown. As is seen in Fig. 5 (c), the difference $X^{Given}(t)-X^{GP}(t)$ in the two series for X(t) becomes too large after around t=115, as is expected for a chaotic time series. The GP generated parametric plot in Fig. 5 (d) matches remarkably well with the parametric plot for original Rossler system as shown in Fig. 4 (b).

5.3 HENON MAP

Finally, consider the Henon Map having two nonlinear coupled map equations as shown in Equation (5),

$$X_{n+1} = Y_n + 1 - \alpha X_n^{\ 2} \qquad (5)$$
$$Y_{n+1} = \beta X_n$$

The Henon equations are iterated with the system parameters (α, β) = (1.4, 3) and the initial conditions as $X_0=Y_0=1.0$, and the plots obtained after removing some initial transient points are shown in Fig. 6 corresponding to X_n (Fig. 6 (a)) and the parametric plot for X_n versus Y_n (Fig. 6 (b)).

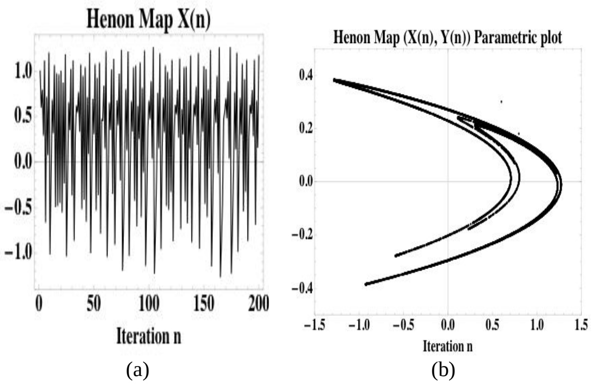


Figure 6 (a) The map for X_n and (b) parametric plot for X_n versus Y_n for Henon Map.

Using the data generated by the map equation in Equation (5), the enhanced GP method is used for the inference of system equations for Henon map. As regards the blind spot effect, it is observed that for the Y_{n+1} equation there is just one term on the right hand side of the equation, and consequently the blind spot effect is not observed. On the other hand, for the X_{n+1} equation, the effect is clearly seen around the chromosome **1 -1.4X$_n^2$** having fitness value 0.8856, as shown in Table 5.

Table 5: Chromosomes and their fitness values generated during GP experiments for Henon Map equations showing the occurrence of blind spot effect.

Equation $X_{n+1} = Y_n + 1 - 1.4 X_n^2$			
Fitness	Chromosome	Fitness	Chromosome
0.9714	**1-1.4X$_n^2$** $+ 0.5Y_n$	0.9034	**1-1.4X$_n^2$** $+ X_n^3Y_n$
0.9076	**1-1.4X$_n^2$** $+ 0.2X_n^2Y_n$	0.9026	**1-1.4X$_n^2$** $+ X_n^2Y_n^2$
0.9034	**1-1.4X$_n^2$** $- X_nY_n$	0.8997	**1-1.4X$_n^2$** $+ Y_n^2$
0.8903	**1-1.4X$_n^2$** $-X_nY_n-Y_n^2$	0.8961	**1-1.4X$_n^2$** $+ X_n^3Y_n$
0.8856	**1-1.4X$_n^2$**		

Resolving the blind spot effect as described earlier in section 4.1, the enhanced GP method is successful for the inference of the map equations with an accuracy of six significant digits, and the inferred equations have the same symbolic structure as in Equation (5), with the system parameters as $(\alpha, \beta)_{GP} = (1.399992, 0.299994)$.

The plots for Henon map as generated using the GP inferred equations are shown in Fig. 7. The difference between X(n) series as given by original equation (5) and as given by GP inferred equations is shown in Fig. 7 (c), and it is seen that the difference goes hay wire after around 25 iterations, as per the characteristics of a chaotic solution. However the parametric plot as given in Fig. 7 (d) shows a remarkable similarity with the plot (Fig. 6 (b)) as given by the original Henon equations.

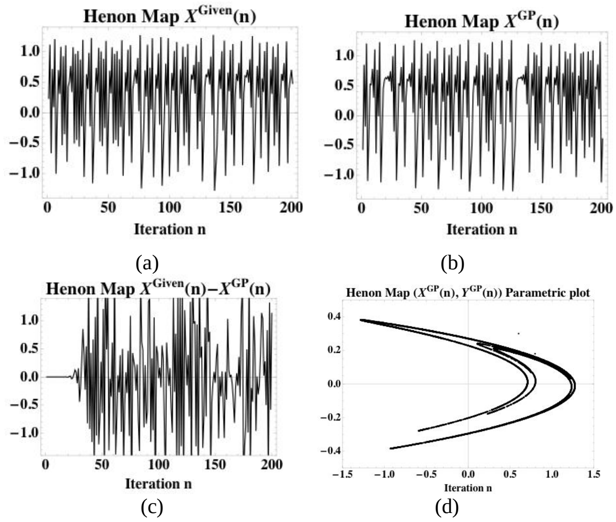


Figure 7 The plots for Henon map: (a) original Henon map series $X^{Given}(t)$, (b) GP inferred Henon map series $X^{GP}(t)$, (c) the difference in two series $X^{Given}(t)-X^{GP}(t)$ and (d) parametric plot $X^{GP}(t)$ versus $Z^{GP}(t)$.

6. CONCLUSION AND OUTLOOK

In the present paper inference of dynamical model equations in terms of coupled nonlinear ODE equations for chaotic time series data has been carried out quite successfully for a number of well known chaotic systems, namely Lorenz, Rossler and Henon map systems using the enhanced search based GP method. The important aspect reported in the paper is the observation of an interesting effect, we call as a Blind Spot Effect, which is attributed to one of the reasons why a standard GP based search method for inference of coupled ODE equations may become difficult and challenging. Routines have been implemented to deal with the effect successfully. In continuation with the present results, work is in progress to identify possible occurrence of the effect in some other physical systems, including non-linear Schrodinger equation, Korteweg-de Vries equation etc that are known to have soliton solutions.

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