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Digital signal processing: Roles of Z-transform & Digital Filters

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Abstract

This Research Paper first, briefly explains Z-transform, compares it with Laplace transform and also briefly explains difference equation and differential equation.it then takes up difference equation and demystifies its implementation in digital filtering operations. This work further x-rays digital filters with focus on finite impulse response (FIR) and Infinite impulse Response (IIR) filter, basic comparison between IIR and FIR are also highlighted.

Finally, based on this work, choice of a filter for a particular purpose is hinted. However, this work does not go into mathematical derivations but it did use the relevant formulae and equations as it pertains to digital signal processing and viewed from relevant application perspectives.

Keywords: Z-transform, difference equation, Finite Impulse Response (FIR), Infinite Impulse Response (IIR).

1. INTRODUCTION

With the explosion of digital communication and media, the need for methods to analyze and process digital data is becoming more important than ever. JPEG images, MP3 & MP4 songs, MPEG-2 videos, and ZIP files etc. are all processed using digital processing techniques. According to Professor. Todd K. Moon, Z-transform like the Laplace transform, is an indispensable mathematical tool for the design, analysis and monitoring the stability of a system.

Z-transform converts a sequence, {x[n]}, into a function, X(z), of an arbitrary complex-valued variable z.This is very important as complex functions are easy to manipulate than sequence. Mathematical descriptive relationship of input-output of a system is formulated either in the time domain or in the frequency domain. Time-domain and frequency domain representation methods offer alternative clear insights into a system, and depending on the application, it may be more convenient to use one method in preference to the other. Time domain system analysis methods are based on differential equations which describe the system output as a weighted combination of the differentials (i.e. the rates of change) of the system input and output signals. Frequency domain methods, mainly the Laplace transform, the Fourier transform, and the z-transform, describe a system in terms of its response to the individual frequency constituents of the input signal. This research reveals that the Z-transform determines the stability of a system as description and representation of a system in the frequency domain reveals valuable insight into the system behavior and stability. In addition to this, the transient and the steady state characteristics of a system can be predicted by analyzing the roots of the z-transform, which are so-called poles and zeros of a system. Hence Z-transform is highly utilized in the digital signal processing and digital filters analysis.

2. BRIEF COMPARISON BETWEEN Z-TRANSFORM AND LAPLACETRANSFORM

Here, it is important to note that Laplace transform methods are widely used for analysis in linear systems. And they are used when a system is described by a linear differential equation, with constant coefficients. However Z-transforms are used in numerous systems that are described by difference equations - not differential equations - and those systems are common and different from those described by differential equations.

Systems that satisfy difference equations include computer controlled systems - systems that take measurements with digital I/O boards, calculate an output voltage and output that voltage digitally. Frequently these systems run a program loop that executes in a fixed interval of time. Other systems that satisfy difference equations are those systems with Digital Filters - which are found anywhere digital signal processing - digital filtering is done. These include digital signal transmission systems like the telephone system and systems that process audio signals. For example, a CD contains digital signal information, and when it is read off the CD, it is initially a digital signal that can be processed with a digital filter. There are an incredible number of systems we use every day that have digital components which satisfy difference equations.

Next is to look at Laplace transforms and Z-transforms under controlled and sampled systems. In continuous systems, Laplace transforms play a unique role. They allow system and circuit designers to analyze systems and predict performance, and to think in different terms - like frequency responses - to help understand linear continuous systems. They are a very powerful tool that shapes how engineers think about those systems. Z-transforms play similar role in sampled systems as

stated below relative to Laplace:

- In continuous systems, inputs and outputs are related by differential equations and Laplace transform techniques are used to solve those differential equations.
- In sampled systems, inputs and outputs are related by difference equations and Z-transform techniques are used to solve those difference equations.
- In continuous systems, Laplace transforms are used to represent systems with transfer functions, while in sampled systems, Z-transforms are used to represent systems with transfer functions.

2.1 What is a sampled system?

Simply put, Digital filters are by essence sampled systems. The input and output signals are represented by samples with equal time distance. A typical sampled system is as depicted in figure 1 below:

Figure 2.1.1 A typical sampled system

- An analog signal is converted to a digital form in an A/D.
- The digital signal is processed somehow.
- The processed digital signal is converted to an analog signal for use in the analog world

3 .DIFFERENTIAL, DIFFERENCE EQUATIONS AND DIGITAL FILTERS

3.1 Differential equation

This is not to be confused with difference equation vice versa. A differential equation is a mathematical equation that relates some function with its derivatives. And their derivatives represent their rates of change, and the equation defines a relationship between the two. The input and Output of a continuous system which are related by differential equation are solved by Laplace transform technique. However, applicable to sampled system are Z-transform techniques which solve the difference equation that forms the relationship between its input and output

3.2 Difference equation

A difference equation is an equation which expresses a relation between an independent variable and the successive values of the dependent variable or the successive differences of the dependent variable. This represents a Linear Time Invariant (LTI) system and obeys all its usual properties. Therefore, difference equation is to discrete signal processing what the differential equation is to analogue signal processing. It is used to describe operation of discrete time system

3.3 Digital Filters

This is a system (sampled system) that performs mathematical operations (Z-transform techniques) on a sampled, discrete-time signal to suppress or boost certain aspects of that signal and it is principally implemented in digital signal processing (DSP) devices. A digital filter system usually consists of an analog-to-digital converter to sample input signal, followed by a microprocessor and some peripheral components such as memory to store data and filter coefficients etc. Finally, a digital-to-analog converter to complete the output stage and Program Instructions (software) running on the microprocessor implemented by the digital filter by performing the necessary mathematical operations on the numbers received from the ADC(as simplified in the above figure 2.1.1). However, it is worthy to note here that analogue filters can yield similar output as digital filters, only that digital filters can achieve far superior results. It is important also that digital filters are characterized by its transfer function, or equivalently, its difference equation as it is the case in this research work.

3.3.1 Difference Equation implementation in Digital filters

The difference equation is a formula for computing an output sample at time □ based on past and present input samples and past output samples in the time domain. Hence the general causal Linear Time Invariant (LTI) difference equation is written as

Where x is the input signal, y is the output signal, and the constants bi, i = 0, 1, 2, ...,M, aj, j = 1, 2, ..., N are called the coefficients. As a specific example, the difference equation: y (□) = 0.03x (□) + 0.004x(□ -1) + 0.89y(□ -1) specifies a digital filtering operation, and the coefficient sets (0.03, 0.004) and (0.89) fully characterize the filter. In this example, we have M = N= 1. When the coefficients are real numbers, as in the above example, the filter is said to be real. Otherwise, it may be complex.

$$= \sum_{i=0}^{M} bi\, x(\eta - i) - \sum_{j=1}^{N} aj\, y(\eta - j)$$

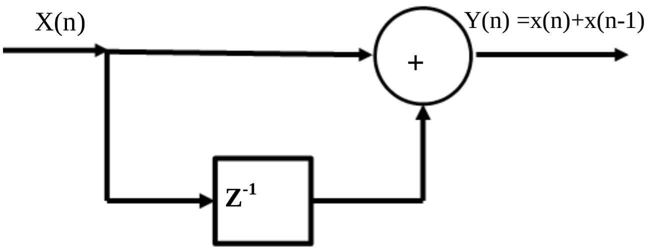
This forms the basis for design of digital filters. With this, digital filters are categorized into two broad classes:

- Recursive
- Non-recursive

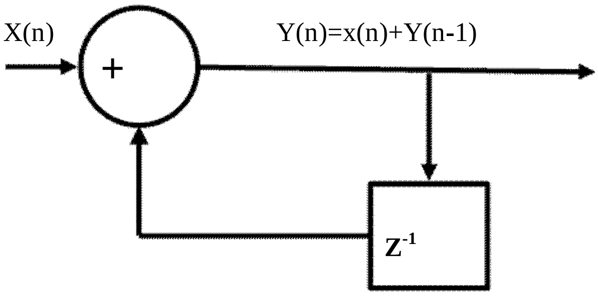
Hence based on above equation, any filter having one or more feedback paths (N>1) is called Recursive. Specifically, the bi coefficients are called the feedforward coefficients and the ai coefficients are called the feedback coefficients. A filter is said to be recursive if and only if ai is not equal to zero for some i > 0. Recursive filters are also called infinite-impulse-response (IIR) filters. It is important to note that Its impulse response of an Nth-order FIR filter lasts for N+ 1samples, and then dies to zero. This means that for a finite number of sample intervals, it finally settles to zero. But when there is no feedback (ai=0, for all i >0), the filter is said to be a non-recursive or finite-impulse-response (FIR) digital filter. It is finite because its impulse response is of finite duration. This is in contrast to recursive - infinite impulse response

(IIR) filters which have internal feedback and may theoretically continue to respond indefinitely. The impulse response of a digital filter is the output sequence from the filter when a unit impulse is applied at its input. (A unit impulse is a very simple input sequence consisting of a single value of 1 at time t = 0, followed by zeros at all subsequent sampling instants).

Therefore Recursive filters are efficient way of achieving a long impulse response, without having to perform a long convolution. They execute very rapidly, but have less performance and flexibility than other digital filters. Recursive filters are useful because they bypass a longer convolution. Since this impulse response is infinitely long, hence recursive filters are called infinite impulse response (IIR) filters. In effect, recursive filters convolve the input signal with a very long filter kernel, although only a few coefficients are involved. Below diagrams illustrates non-recursive and recursive digital filters:



Non- Recursive digital Filter (as adapted from digital signal processing by Zeyad Al-Hamdany)



Recursive digital filter

3.4 Basic Differences between Infinite Impulse Response (IIR) – Recursive and Finite Impulse Response (FIR) – Non-recursive filters

Efficient Memory utilization and throughput: The obvious main difference between IIR filters and FIR filters is that an IIR filter is more compact in that it can usually achieve a prescribed frequency response with a smaller number of coefficients than an FIR filter. A smaller number of filter coefficients imply less storage requirements and faster calculation and a higher throughput. Therefore, generally IIR filters are more efficient in memory and computational requirements than FIR filters.

Stability: FIR filter is always stable, whereas an IIR filter can become unstable and care must be taken in the design of IIR filters to ensure stability. An FIR filter will be stable no matter how it is synthesized or implemented as it has no feedback. On the other hand, an IIR filter with improperly placed poles can't be made stable no matter how it is implemented.

Math Register Size: IIR takes up more register size than FIR during peak signal processing. The peak math value for IIR can range from 1.0 to 10,000 or even higher. A filter's peak math value is a strong function of the polynomial it is based on and its selectivity. The more selective the filter, the higher the math values.

Implementation structures: FIR filters are usually implemented as Nth order polynomials. IIR filters on the other hand, can be implemented this way, but only if a floating point processor is available. If using a fixed point processor, an IIR filter must be implemented as a series of second order. This is to guard against overflow during peak math value which is highly forbidden which can lead to its abnormal behavior

4. CONCLUSION

This research work has thoroughly x-rayed and anatomized the beauties of Z-transform application in solving difference equations which form the bedrock of digital filters operations. It is also emphatic to note that IIR filter is neither totally superior nor worse than FIR filter. Each has its suitability, and the project's overall requirements and cost determine choice.

Therefore, One`s choice for a particular digital filter be it IIR or FIR, does not holistically make the chosen filter the best solution as each filter which is preferred in one solution, maybe least thought of in another. This is because a lot of factors are put into consideration (as discussed above) such as cost, delay or latency, stability etc. in choosing one digital against another.

Other recommendations

Moving forward, this research can be further steered towards mathematical derivations / proofs of the above stated assertions.

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