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AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH AN EXPONENTIALLY DEMAND WITH TIME

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Abstract

In this paper, a deterministic inventory control model with exponentially increase in demand rate with time has been proposed. The demand rate increase by a constant percentage during each time interval. This proposed model will be constrained binomial geometric programming model. This indicates that the larger, the increase in quantity the faster it grows. A numerical analysis of the prepared model has been presented. Production rate is considered as finite and approximation procedure is used to solve the model.

Keywords:-Deterministic Inventory control, Demand rate, Population, exponential growth

1.INTRODUCTION

Various work has been done for determining the inventory level of deteriorating iterns' which allows and does not allow shortage by different researchers over last three decades. Maximum physical goods undergo decay or deterioration over time. Fruits, vegetables and food item suffer from depletion by direct spoilage while stored. Highly volatile liquid such as gasoline alcohol and turpentine undergo physical depletion overtime through the process of evaporation. Electronic goods, radioactive substance, photographic film, grain etc. Deteriorate through a gradual Loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory feature felt the necessity to use this factor into consideration. A number of researchers have developed model in the area of deteriorating inventory model was developed by ghare and schadev (1963)

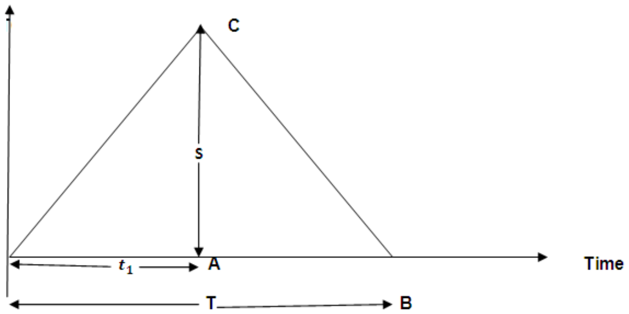
Assumption and Notations

(1) Replenishment rate is finite.
(2) The lead time is zero.
(3) T is fixed duration of each production cycle.
(4) θ is the constant rate of deterioration.
(5) q_t is the inventory level at any time t.
(6) S is the initial inventory after fulfilling back orders.
(7) D is the total amount of deteriorated units.
(8) K is the average total cost.
(9) C is the cost of each unit.
(10) C_1 is the holding cost per unit per unit time.
(11) C_2 is the shortage cost per unit per unit time.
(12) Q is the total inventory produced at the beginning of each period.

Mathematical Formulation And Analysis For The System

Let $q(t)$ is the inventory level at any time ($0 \le t \le \pi$). We consider Q as total and S is the initial inventory after fulfilling backorders. Inventory level gradually decreases during time $(0, \frac{\pi}{2})$ due to market demand and deterioration and ultimately fall to zero at $\frac{\pi}{2}$. Shortage occurs during period $(\frac{\pi}{2}, \pi)$which are fully backlogged. The cycle then repeats itself. We have to determine optimal value of S, this model has been shown graphically in figure1.

NVENTORY



The differential equation governing the system in time interval $(0, \pi)$ are given by

$$\frac{dq(t)}{dt} + \theta q(t) = -\alpha \sin t \qquad 0 \le t \le \frac{\pi}{2} \qquad \ldots\ldots\ldots(1)$$

$$\frac{dq(t)}{dt} = -\alpha \sin t \qquad \frac{\pi}{2} \le t \le \pi \qquad \ldots\ldots\ldots(2)$$

Equation (1) is a linear differential equation.
Its solutions is given by

$$q(t)e^{\theta t} = -\int \alpha \sin t^{\theta t}\, dt + c$$

$$q(t)e^{\theta t} = \frac{\alpha}{1+\theta^2}[\cos t - \theta \sin t] + C$$

$$q(t) = \frac{\alpha e^{-\theta t}}{1+\theta^2}[\cos t - \theta \sin t] + c.e^{-\theta t} \qquad (3)$$

where c is the constant of integration. Solution of equation(2) is

$$q(t) = -a \int \sin dt + c$$

$$q(t) = a \cos t + c \quad (4)$$

where c is the constant of integration.
Applying the boundary condition q(1) = S at $t = 0$ in equation (3) we get

$$S = \frac{\alpha}{1+\theta^2}[1] + c$$

$$c = S - \frac{\alpha}{1+\theta^2}$$

equation (3) is reduces to

$$q(t) = \frac{\alpha e^{-\theta t}}{1+\theta^2}[\cos t - \theta \sin t] + \left(S - \frac{\alpha}{1+\theta^2}\right).e^{-\theta t}$$

$$q(t) = Se^{-\theta t} + \frac{\alpha e^{-\theta t}}{1+\theta^2}[\cos t - \theta \sin t] - \frac{\alpha}{1+\theta^2}e^{-\theta t} \quad (5)$$

applying the boundary condition at t = $\frac{\pi}{2}$ at $q(t) = 0$ in equation (4) we get

0=0+c

Or c = 0

Therefore

$$q(t) = \alpha \cos t \qquad \frac{\pi}{2} \le t \le \pi$$

now since at t = $\frac{\pi}{2}$ $q(t) = 0$

so equation (5) reduces to

$$0 = Se^{-\theta\frac{\pi}{2}} + \frac{\alpha e^{-\theta\frac{\pi}{2}}}{1+\theta^2}\left[-\frac{\pi}{2}\right] - \frac{\alpha}{1+\theta^2} e^{-\theta\frac{\pi}{2}}$$

Therefore,

$$s = \frac{\alpha}{1+\theta^2}\left[1+\frac{\pi}{2}\right]$$

$$s = \frac{\alpha(2+\pi)}{2(1+\theta^2)}$$

Hence deteriorated amount of inventory will be

$$D = S - \int_0^{\frac{\pi}{2}} \text{sint } dt$$

$$= S + \alpha$$

$$= \alpha\left[\frac{\alpha(2+\pi)}{2(1+\theta^2)} + 1\right]$$

$$= \frac{\alpha}{2}\left[\frac{(4+\pi)+2\theta^2}{2(1+\theta^2)}\right] \qquad (8)$$

Total average number of units in inventory during (0, $\frac{\pi}{2}$)

$$q_{1(t_1)} = \frac{1}{T}\left[\int_0^{\frac{\pi}{2}} q(t)dt\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{T}\left[\frac{Se^{-\theta t}}{-\theta} + \frac{a}{(1+\theta^2)}\left\{\frac{a^{-\theta t}}{(1+\theta^2)}[\sin t - \theta\cos t] + \frac{\theta e^{-\theta t}}{(1+\theta^2)}(\cos t - \theta \sin t)\right\} + \frac{ae^{-\theta t}}{\theta(1+\theta^2)}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{T}\left[\frac{se^{-\theta t}}{-\theta} + \frac{ae^{-\theta t}}{(1+\theta^2)^2}[(1+\theta^2)\sin t] + \frac{ae^{-\theta t}}{\theta(1+\theta^2)^2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{T}\left[\frac{se^{-\theta t}}{-\theta} + \frac{ae^{-\theta t}}{(1+\theta^2)^2}[\sin t] + \frac{ae^{-\theta t}}{\theta(1+\theta^2)^2}\right]_0^{\frac{\pi}{2}}$$

Using equation (7) above equation become

$$q1(t_1) = \frac{1}{T}\left[\frac{\alpha(2+\pi)e^{-\theta t}}{2(1+\theta^2)-\theta} + \frac{ae^{-\theta t}}{(1+\theta^2)^2}[\sin t] + \frac{ae^{-\theta t}}{\theta(1+\theta^2)^2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ae^{-\theta t}}{T(1+\theta^2)}\left[\sin t + \frac{1}{0} - \frac{(2+\pi)}{2\theta}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ae^{-\theta t}}{T(1+\theta^2)}\left[\frac{2\theta \sin t + 2 - 2 - \pi}{2\theta}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{ae^{-\theta t}(2\theta \sin t - \pi)}{2T(1+\theta^2)^{\theta}} \qquad (9)$$

Also average number of units in shortage during $\left(\frac{\pi}{2}, \pi\right)$ are

$$q_{2(t_1)} = \frac{1}{T}\int_{\frac{\pi}{2}}^{\pi} q(t)\,dt \qquad = \frac{1}{T}\int_{\frac{\pi}{2}}^{\pi} a \cos t\,dt$$

$$= \frac{1}{T}[\alpha \sin t]$$

$$= \frac{1}{T}\left[\sin\pi - \sin\frac{\pi}{2}\right] \qquad = \frac{-\alpha}{T} \qquad (10)$$

In equation (10) negative sign shows the shortage.

Finally average total cost per unit time is given by

$$K = \frac{ae^{-\theta t}(2\theta \sin t - \pi)c_1}{2T(1+\theta^2)^{\theta}} - \frac{ac_2}{2} + \frac{ac}{2}\frac{\alpha}{2}\left[\frac{(4+\pi)+2\theta^2}{2(1+\theta^2)}\right] \qquad (11)$$

Here c is the total cost of each unit, C1 is holding cost per unit per unit time,C2 is the shortage cost per unit per unit time,

To get the optimum value of t one need to apply the criteria of optimization as

$$\frac{dk}{dt} = 0$$

$$\tau.e. \frac{ae^{-\theta t}\theta(2\theta \sin t - \pi)C_1}{2T(1+\theta^2)} + \frac{ae^{-\theta t}(2\theta \cos t)C_1}{2T(1+\theta^2)\theta} = 0 \qquad (12)$$

The equation (12) is a transcendental equation. It can be solved numerically to get

optimum

value of t (say t)

Also

$$\frac{d^2k}{dt^2} = \frac{ae^{-\theta t}\theta(2\theta \sin t - \pi)C_1}{2T(1+\theta^2)\theta} - \frac{ae^{-\theta t}\theta(2\theta\cos t)C_1}{2T(1+\theta^2)\theta} + \frac{-2ae^{-\theta t}\theta^2(\cos t)C_1}{2T(1+\theta^2)\theta} - \frac{2ae^{-\theta t}\sin tC_1}{2T(1+\theta^2)\theta}$$

Verify the optimum value of t

Therefore total optimum cost

$$k^* = \frac{ae^{-\theta t^*}\theta(2\theta \sin t^* - \pi)C_1}{2T(1+\theta^2)\theta} - \frac{aC_2}{T} + \frac{aC}{2}\left[\frac{(1+\pi)+2\theta^2}{(1+\theta^2)}\right]$$

Conclusion

In this paper, an order level production inventory model for deteriorating item is presented. The demand rate increase exponentially with time. In this, the production rate depends on demand. The approximate9 expression for holding cost deterioration cost and total cost of the system of obtained. Cost minimization technique is used to get the solution.

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