Adjoint Operators of Fourier-Finite Mellin Transform

V. D. Sharma [1] ,A. N. Rangari [2]

[1] Mathematics Department, Arts, Commerce and Science College, Amravati- 444606(M.S), India.

[2] Mathematics Department, Adarsh College, Dhamangaon Rly.- 444709 (M.S), India.

Abstract

Transform analysis of generalized functions concentrates on finite parts of integrals, generalized function and distributions. The Fourier- Mellin transform (FMT) of an input function is defined as and is the magnitude squared of the Mellin transform of the magnitude squared of the Fourier transform of the input function. A specific form of the Mellin transform, referred to as the "scale transform", is known to be a natural complement to the Fourier transform for wideband analytic signals. While the Fourier-Finite Mellin has found numerous applications in optical pattern recognition, ship classification by sonar and radar and image processing. These Fourier and Finite Mellin transforms have various properties and these properties have various applications in many fields.

The main purpose of this paper is to describe the Adjoint operators of Fourier-Finite Mellin transform.

Keywords: Fourier transform, Finite Mellin transform, Fourier-Finite Mellin transform, Generalized function, Adjoint Operator.

1. INTRODUCTION

The classical theory of integral transformations has been extended to generalized functions by many people. But the main credit goes to Zemanian [1], [2] who gave the way for the extension and called it the theory of generalized integral transformations [3]. Integral transforms provide a way to solve otherwise intractable physical problems. They work by expressing the equations of a physical system in a new form that can be solved with simple computation [4].

Human face recognition is, indeed a challenging task, especially under illumination and pose variation so the effectiveness of a simple face recognition algorithm based on Fourier-Finite Mellin transform [5]. Recently the Fourier-Mellin transform has seen a revival with the advent of watermarking. It also useful in agriculture. Robbins and Huang described an implementation for the application of the Fourier-Finite Mellin transform to correct various optical distortions, including noise, in lenses. In this way there are various application of Fourier-Finite Mellin transform [6]. The term "Operator" is another term for function, mapping or transformation. An operator assigns an object from one set (the co-domain) to an object from another set (the domain).

This paper gives the generalization of Fourier-Finite Mellin transform in the distributional sense by defining the definition of distributional generalized Fourier-Finite Mellin Transform which is as follows:

$$FM_f\{f(t,x)\} = F(s,p) = \left\langle f(t,x), e^{-ist}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle \quad (1.1.1)$$

where, for each fixed t $(0<t<\infty)$, x $(0<x<\infty)$, $s>0$ and $p>0$, the right hand side of (1.1.1) has a sense as an application of $f(t,x) \in FM^{*\beta}_{f,b,c,\alpha}$ to

$$e^{-ist}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right) \in FM^{\beta}_{f,b,c,\alpha} .$$

Also to develop the work we defined some testing function spaces as follows:

a. The space $FM_{f,b,c,\alpha}$

This space is given by

$$FM_{f,b,c,\alpha} = \{\phi : \phi \in E_+ / \xi_{b,c,k,q,l}\phi(t,x)$$

$$\left.\begin{array}{l}\sup \\ = 0<t<\infty \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x)\right| \le C_{lq} A^k k^{k\alpha} \\ 0<x<a\end{array}\right\} \quad (1.2.1)$$

for each $k,l,q = 0,1,2,3,........$ where,

$$\lambda_{b,c}(x) = \begin{cases} x^{+b} & 0<x<1 \\ x^{+c} & 1<x<a \end{cases}$$

Where the constants A and C_{lq} depend on the testing function ϕ.

b. The space $FM_{f,b,c,\gamma}$

It is given by

$$FM_{f,b,c,\gamma} = \{\phi : \phi \in E_+ / \gamma_{b,c,k,q,l}\phi(t,x)$$

$$\left.\begin{array}{l}\sup \\ = 0<t<\infty \left|t^k \lambda_{b,c}(x) x^{q+1} D_t^l D_x^q \phi(t,x)\right| \le C_{lk} A^q q^{q\gamma} \\ 0<x<a\end{array}\right\} \quad (1.3.1)$$

Where, $k,l,q = 0,1,2,3,........$ and the constants depend on the testing function ϕ.

The main aim of this paper is to described some Adjoint Operators of Fourier-Finite Mellin transforms. The plan of this paper is as follows:

In section 2, Adjoint shifting-scaling operator for Fourier-Finite Mellin transform is defined, Adjoint diffetrential operators of Fourier- Finite Mellin transforms is described in section 3. section 4 concludes the paper.
Notations and terminology as per Zemanian. [1], [2].

2. ADJOINT OPERATORS OF FOURIER-FINITE MELLIN TRANSFORM

2.1 Theorem:
The adjoint shifting operator is a continuous function from $FM^*_{f,b,c,\alpha}$ to $FM^*_{f,b,c,\alpha}$.The adjoint operator $f(t,x)\to f(t-\tau,x)$ leads to the operation transform formula

$$FM_f\{f(t-\tau,x)\}=e^{-is\tau}FM_f\{f(t,x)\}$$

Proof: - Consider,

$$FM_f\{f(t-\tau,x)\}=\left\langle f(t-\tau,x)e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=\left\langle f(t,x),e^{-is(t+\tau)}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=e^{-is\tau}\left\langle f(t,x),e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=e^{-is\tau}FM_f\{f(t,x)\}$$

$$\therefore FM_f\{f(t-\tau,x)\}=e^{-is\tau}FM_f\{f(t,x)\}$$

2.2 Theorem:
The adjoint scaling operator is a continuous function from $FM^*_{f,b,c,\alpha}$ to $FM^*_{f,b,c,\alpha}$. The adjoint operator $f(t,x)\to\frac{1}{q}f\left(t,\frac{x}{q}\right)$ corresponding transform formula is

$$FM_f\left\{\frac{1}{q}f\left(t,\frac{x}{q}\right)\right\}=QFM_f\{f(t,x)\}$$

Proof:-Consider,

$$FM_f\left\{\frac{1}{q}f\left(t,\frac{x}{q}\right)\right\}=\left\langle\frac{1}{q}f\left(t,\frac{x}{q}\right),e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=\left\langle f(t,x),e^{-ist}\left(\frac{a^{2p}}{(qx)^{p+1}}-(qx)^{p-1}\right)\right\rangle$$

$$=\left\langle f\left(t,\frac{X}{q}\right),e^{-ist}\left(\frac{a^{2p}}{X^{p+1}}-X^{p-1}\right)\right\rangle$$ Where

$$qx=X\ ,\quad x=\frac{X}{q}$$

$$=Q\left\langle f(t,x),e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$, where Q is constant depending on q.

$$=QFM_f\{f(t,x)\}$$

$$\therefore FM_f\left\{\frac{1}{q}f\left(t,\frac{x}{q}\right)\right\}=QFM_f\{f(t,x)\}$$

2.3 Proposition:

The adjoint shifting-scaling operator is a continuous function from $FM^*_{f,b,c,\alpha}$ to $FM^*_{f,b,c,\alpha}$. The adjoint operator $f(t,x)\to\frac{1}{q}f\left(t-\tau,\frac{x}{q}\right)$. Correspondingly we can prove

$$FM_f\left\{\frac{1}{q}f\left(t-\tau,\frac{x}{q}\right)\right\}=e^{-is\tau}QFM_f\{f(t,x)\}.$$

Note that Fourier-Finite Mellin transform is shift-scale invariant.

3. ADJOINT DIFFERENTIAL OPERATORS OF FOURIER-FINITE MELLIN TRANSFORM

3.1 Theorem:

The adjoint differential operator $f\to D_t f$ is continuous linear mapping from the dual space $FM^*_{f\,f,b,c,\alpha}$ into itself. Corresponding transform formula is

$$FM_f\{D_t f(t,x)\}=(is)FM_f\{f(t,x)\}$$

Proof: Consider,

$$FM_f\{D_t f(t,x)\}=\left\langle D_t f(t,x),e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=\left\langle f(t,x),-D_t e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=\left\langle f(t,x),(is)e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$=(is)\left\langle f(t,x),e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\right\rangle$$

$$\therefore FM_f\{D_t f(t,x)\}=(is)FM_f\{f(t,x)\}$$

3.2. Theorem:

The adjoint differential operator $f\to xD_x f$ is continuous linear mapping from the dual space

$FM^{*}_{f,b,c,\alpha}$ into itself. Corresponding transform formula is $FM_f\left\{xD_x f\left(t,x\right)\right\} = pFM_{f_2}\left\{f\left(t,x\right)\right\}$

Proof: Consider,

$$FM_f\left\{xD_x f\left(t,x\right)\right\} = \left\langle xD_x f\left(t,x\right), e^{-ist}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), -D_x e^{-ist} x\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), -D_x e^{-ist}\left(a^{2p}x^{-p} - x^{p}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), -e^{-ist}\left(a^{2p}\left(-p\right)x^{-p-1} - px^{p-1}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), pe^{-ist}\left(a^{2p}x^{-p-1} + x^{p-1}\right)\right\rangle$$

$$= p\left\langle f\left(t,x\right), e^{-ist}\left(\frac{a^{2p}}{x^{p+1}} + x^{p-1}\right)\right\rangle$$

$= pFM_{f_2}\left\{f\left(t,x\right)\right\}$,where FM_{f_2} is second type of Fourier-Finite Mellin transform.

$$FM_f\left\{xD_x f\left(t,x\right)\right\} = pFM_{f_2}\left\{f\left(t,x\right)\right\}$$

3.3 Theorem

The adjoint differential operator $f \to x^2 D_{xx} f$ s a continuous linear mapping from the dual space $FM^{*}_{f,b,c,\alpha}$ into itself, corresponding operator transform formula is

$$FM_f\left\{x^2 D_{xx} f\left(t,x\right)\right\} = pFM_{f_2}\left\{f\left(t,x\right)\right\}$$

Proof: Consider,

$$FM_f\left\{x^2 D_{xx} f\left(t,x\right)\right\} = \left\langle x^2 D_{xx} f\left(t,x\right), e^{-ist}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), -D_{xx} e^{-ist} x^2\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), -xD_{xx} e^{-ist}\left(a^{2p}x^{-p} - x^{p}\right)\right\rangle$$

$$= \left\langle f\left(t,x\right), -xD_x e^{-ist}\left(a^{2p}\left(-p\right)x^{-p-1} - px^{p-1}\right)\right\rangle$$

$$= p\left\langle f\left(t,x\right), xD_x e^{-ist}\left(a^{2p}x^{-p-1} + x^{p-1}\right)\right\rangle$$

$$= p\left\langle f\left(t,x\right), D_x e^{-ist}\left(a^{2p}x^{-p} + x^{p}\right)\right\rangle$$

$$= p\left\langle f\left(t,x\right), e^{-ist}\left(a^{2p}\left(-p\right)x^{-p-1} + px^{p-1}\right)\right\rangle$$

$$= -p^2\left\langle f\left(t,x\right), e^{-ist}\left(\frac{a^{2p}}{x^{p+1}} - x^{p-1}\right)\right\rangle$$

$= -p^2 FM_{f_1}\left\{f\left(t,x\right)\right\}$, where FM_{f_2} is second type of Fourier-Finite Mellin transform.

$$FM_f\left\{x^2 D_{xx} f\left(t,x\right)\right\} = -p^2 FM_{f_1}\left\{f\left(t,x\right)\right\}$$

4. CONCLUSION

This paper gives the Generalization of Fourier-Finite Mellin transform in the distributional sense. And some Adjoint Operators of Fourier-Finite Mellin transform along with the properties of Fourier-Finite Mellin transform are defined, which will be useful when this transform will be used to solve differential and integral equations.

References

[1] A. H. Zemanian, "Generalized integral transform," Inter science publisher, New York, 1968.

[2] A.H. Zemanian, "Distribution theory and transform analysis," McGraw Hill, New York, 1965.

[3] V.D. Sharma, and A.N. Rangari, "Introduction to Fourier-Finite Mellin transform," European Journal of Applied Engineering and Scientific Research, 3(3), pp. 43-48, (2014).

[4] Berian J. James, "Integral transforms for you and me," Royal Observatory Edinburgh Institute for Astronomy, March 2008.

[5] Sambhunath Biswas, and Amrita Biswas, "Fourier Mellin Transform based face recognition," Int. Journal of Computer engineering and technology (IJCET), Vol.4, issue 1, pp. 8-15, 2013.

[6] V.D. Sharma, and A.N. Rangari, "Operational Calculus on Fourier-Finite Mellin Transform," Int. Journal of Engineering and Innovative Technology (IJEIT), 3(4), pp. 161-164, (2013).

[7] V. I. Agoshkov, and P. B. Dubovski, "Methods of Integral Transforms," Computational Methods and Algorithms, Vol. I.

[8] M.C. Anumaka, "Analysis and applications of Laplace/Fourier transformations in electric circuit," IJRRAS, 12(2), August 2012.

[9] Lokenath Debnath and Dambaru Bhatta, "Integral Transforms and their Applications," Chapman and Hall/CRC Taylor and Francis Group Boca Raton London, New York, 2007.

[10] Anupama Gupta, "Fourier Transform and Its Application in Cell Phones," International Journal of Scientific and Research Publications, Volume 3, Issue 1, pp. 1-2 January 2013.

[11] V.D. Sharma, "Operation Transform Formulae on Generalized Fractional Fourier Transform," Proceedings International Journal of Computer Applications (IJCA), (0975-8887), PP. 19-22, 2012.

[12] V.D. Sharma, and A.N. Rangari, "Operation Transform Formulae of Fourier-Laplace Transform," Int. Journal of Pure and Applied Sciences and Technology, 15(2), pp. 62-67, (2013).

[13] V.D. Sharma, and A.N. Rangari, "Operational Calculus on Generalized Fourier-Laplace Transform," Int. Journal of Scientific and Innovative Mathematical Research (IJSIMR), 2(11), pp. 862-867, (2014).

[14] V.D. Sharma, and A.N. Rangari, "Properties of Generalized Fourier-Laplace Transform," Int. Journal of Mathematical Archive (IJMA), 5(8), pp. 36-40, 2014.

[15] R. J. Beerends, H. G. ter Morsche, J. C. van den Berg, and E. M. van de Vrie, "Fourier and Laplace Transforms," Cambridge University Press, 2003.

[16] Patrick Fitzsimmons and Tucker Mc EL Roy, "On Joint Fourier-Laplace Transforms," Communication in statistics-Theory and Methods, 39: 1883-1885 Taylor and Francis Group, LLC, 2010.

AUTHOR

Dr. V. D. Sharma is currently working as an Assistant professor in the department of Mathematics, Arts, Commerce and Science College, Kiran Nagar, Amravati-444606 (M.S.) India. She has obtained her Ph.D. degree in 2007 from SGB Amravati University Amravati. She has got 16 years of teaching and research experience. Her field of interest is Integral Transforms. She has published more than 70 research articles. Six research students are working under her supervision.

A. N. Rangari is an Assistant professor in the department of Mathematics, Adarsh College, Dhamangaon Rly., Dist: Amravati-444709 (M.S.) India. She has got 8 years of teaching experience. She has obtained her master degree in 2006 and M.Phil.degree in 2008 from RTM Nagpur University, Nagpur. She has 15 research articles in International Journals to her credit.