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Vector optimization in the system optimal Decision-making in economic and technical systems

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Abstract

The purpose of work to give theoretical justification and to formulate methods of the solution of the vector problems of mathematical programming (VPMP) at equivalent criteria. Methods of the solution of VPMP are based on normalization of criteria and the principle of the guaranteed result. Methods of vector optimization are intended for system adoption of optimal solutions, first, for modeling and adoption of optimal solutions in economic systems, and, secondly, for modeling and adoption of optimal solutions in technical systems in the conditions of definiteness and uncertainty. When modeling economic systems there is a problem of the choice of the optimum choice of goods at which their production is estimated by system of economic indicators. The model of economic system is presented by a vector problem of linear programming. The model of technical system is created as a nonlinear vector problem of mathematical programming. Criteria (characteristics) are formed in the conditions of definiteness (functional dependence of each characteristic and restrictions on parameters is known) and in the conditions of uncertainty (there is no sufficient information on functional dependence of each characteristic on parameters). Numerical implementation of methods of the solution of problems of vector optimization is shown: in economic sciences on an example with a vector problem of linear programming with three criteria; in technical science it is shown on a numerical example of a vector problem of nonlinear programming with five criteria (characteristics). Both tasks are realized in the Matlab system.

Keywords: The Vector optimization, Economic systems, Technical systems, System optimum decision-making

1. INTRODUCTION

At the beginning of the 20th century V. Pareto, [1] at research of a commodity exchange has formulated criterion of optimality, for the purpose of an assessment given criterion improves the general welfare in economy. Pareto's criterion claims that any change which causes nobody losses and which brings to some people benefit is improvement. It is the first example of system (multi-purpose) recommendations about decision-making in economic systems.

Further Pareto's criterion has been postponed for optimizing problems with a set of criteria. In this problem optimization which means that improvement of one or several indicators (criteria) is possible provided that other criteria didn't worsen is considered. There were so multicriteria problems of optimization. A set of criteria in such problems, as a rule, present in the form of a vector of criteria. From there was a name - problems of vector optimization or the vector problems of mathematical programming (VPMP).

The solution of the problem of vector optimization is caused by a number of difficulties, and conceptual character. The main problem is as follows: "That means to solve a problem of vector optimization". For this purpose it is necessary to create the principle of an optimality which shows why one decision is better than another. Such principle defines the rule of the choice of the best decision.

Vector optimization is used in problems of modeling of economic systems: enterprises [15, 19, 20], markets [15, 16, 18], regional economies [15, 17, 18]. To a problem of mathematical modeling of technical systems as much attention is paid to a component of the computer-aided engineering system, as in Russia [1-13], and abroad in theoretical [22, 24, 25] and applied aspects [23, 26-28]. Vector optimization as the device of system decision-making it is presented in works [11, 12, 13]. This work is directed to research and development of methods of system adoption of optimal solutions.

Article purpose – theoretical justification (axiomatics) and the formulation of methods of the solution of problems of vector optimization at equivalent criteria. Methods of vector optimization are intended for system adoption of optimal solutions, first, for modeling and optimal decision making in economic systems, and, secondly, for modeling and decision making in technical systems in the conditions of definiteness and uncertainty.

For realization of a goal in work the following problems are considered and solved:

We have presented theoretical justification of the solution of problems of vector optimization in the form of a number of axioms; methods on the basis of normalization of criteria, the principle of the guaranteed result are developed; methods of the solution of problems of vector optimization are intended for system adoption of optimal solutions.

When modeling economic systems there is a problem of the choice of the optimum choice of goods at which their production is estimated by system of economic indicators. The model of economic system is presented by a vector problem of linear programming.
The model of technical system is formulated in the form of a vector problem of mathematical programming. On the basis of the developed model we conduct research, modeling and we choose optimum parameters of technical systems. Such direction of researches considerably reduces terms of design and increases quality of the created technical systems. The choice of optimum parameters on some system of characteristics it is shown on a numerical example with five criteria (characteristics), realized in the Matlab system.

2. Vector optimization with equivalent criteria

2.1 The problem vector optimization

The vector problem of mathematical programming is the standard problem of mathematical programming having some set of criteria which in total represent a vector of criteria. Distinguish uniform and non-uniform VPMP.
Uniform VPMP of maximizing (minimization) is a vector task which each criterion is directed to maximizing (minimization). *Non-uniform VPMP* are a vector task at which the set of criteria is shared into two subsets (vector) of criteria – maximizing and minimization respectively, i.e. non-uniform VPMP are an association of two types of uniform tasks. According to these definitions we will present a vector problem of mathematical programming with non-uniform criteria [4, 7, 8-10, 14] in the form:

$$Opt\ F(X) = \{max\ F_1(X) = \{max\ f_k(X),\ k = \overline{1, K_1}\ , \qquad (1)$$

$$min\ F_2(X) = \{min\ f_k(X),\ k = \overline{1, K_2}\ \}, \qquad (2)$$

$$G(X) \le B,\ X \ge 0, \qquad (3)$$

where $X=\{x_j,\ j=\overline{1, N}\ \}$ - a vector of material variables, N-dimensional Euclidean space of R^N, (designation $j=\overline{1, N}$ is equivalent to j=1,...,N);
$F(X)$ - a vector function (vector criterion), $F(X)=\{f_k(X),\ k=\overline{1, K}\ \}$. The set $\boldsymbol{K}$ consists of sets of $\boldsymbol{K}_1$ a component of maximizing and $\boldsymbol{K}_2$ of minimization; $\boldsymbol{K}=\boldsymbol{K}_1\cup\boldsymbol{K}_2$ therefore we enter the designation the operation "*opt*" including *max* and *min*; $F_1(X) = \{f_k(X),\ k=\overline{1, K_1}\ \}$ - vector criterion of maximizing, K_1 – number of criteria, and $\boldsymbol{K}_1\equiv\overline{1, K_1}$ - a set of indexes of criterion; $\boldsymbol{K}_2\equiv\overline{K_1+1, K}\equiv\overline{1, K_2}$ - vector criterion of minimization. $\boldsymbol{K}_1\cup\boldsymbol{K}_2 = \boldsymbol{K},\ \boldsymbol{K}_1\subset\boldsymbol{K},\ \boldsymbol{K}_2\subset\boldsymbol{K}$.

$G(X) \le B,\ X\ge 0$ - standard restrictions, $g_i(X) \le b_i$, i=1,...,M where b_i - a set of real numbers, and gi(X) are assumed continuous and convex.

$\boldsymbol{S} = \{X\in\boldsymbol{R}^N \mid X\ge 0,\ G(X) \le B\}\neq\emptyset$ - the set of admissible points set by restrictions (3) isn't empty and represents a compact.

2.2 Axiomatic of vector optimization with equivalent criteria

The At present, theoretical studies and methods of solving vector optimization problems are held in the following directions - methods of solving vector problems based on criteria convolution; methods using restrictions on criteria; goal programming methods; methods based on searching for compromise decision and on human-machine decision making procedures. To analyze the listed methods, we compare the results of solving the test example by these methods with the method based on criteria normalization and the principle of guaranteed result [4, pp. 9-15].

Definition 1. (*Definition of a relative assessment of criterion*).
In a vector problem (1)-(3) we will enter designation:

$$\lambda_k(X) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}\ ,\ \forall k \in \boldsymbol{K}$$

$\lambda_k(X)$ is the relative estimate of a point $X\in S$ k-th criterion;
f_k(X) - k-th criterion at the point $X\in S$; f_k^* - value of the k-th criterion at the point of optimum X_k^* , obtained in vector problem (1)-(3) of individual k-th criterion; f_k^0 is the worst value of the k-th criterion (antioptimum) at the point X_k^0 (Superscript 0 - zero) on the admissible set $\boldsymbol{S}$ in vector problem (1)-(3); the task at *max* (1), (3) the value of f_k^0 is the lowest value of the k-th criterion $f_k^0 = \min_{X\in S} f_k(X)$ $\forall k\in\boldsymbol{K}_1$ and task min f_k^0 is the greatest: $f_k^0 = \max_{X\in S} f_k(X)$ $\forall k\in\boldsymbol{K}_2$. The relative estimate of the λ_k(X), $\forall k\in K$ is first, measured in relative units; secondly, the relative assessment of the $\lambda_k(X)$ $\forall k\in K$ on the admissible set is changed from zero in a point of X_k^0: $\forall k\in\boldsymbol{K}$ $\lim_{X\to X_k^o}\lambda_k(X)=0$, to the unit at the point of an optimum of X_k^*: $\forall k\in\boldsymbol{K}$ $\lim_{X\to X_k^*}\lambda_k(X)=1$, i.e. $\forall k\in\boldsymbol{K}$ $0\le\lambda_k(X)\le 1,\ X\in S$.
This allows the comparison criteria, measured in relative units, among themselves by joint optimization.

Axiom 1. (*About equality and equivalence of criteria in an admissible point of vector problems of mathematical programming*)
In of vector problems of mathematical programming two criteria with the indexes $k\in\boldsymbol{K},\ q\in\boldsymbol{K}$ shall be considered as equal in $X\in\boldsymbol{S}$ point if relative estimates on k-th and q-th to criterion are equal among themselves in this point, i.e. $\lambda_k(X)=\lambda_q(X),\ k,\ q\in\boldsymbol{K}$. We will consider criteria equivalent

in vector problems of mathematical programming if in $X \in S$ point when comparing in the numerical size of relative estimates of $\lambda_k(X)$, $k=\overline{1,K}$, among themselves, on each criterion of $f_k(X)$, $k=\overline{1,K}$, and, respectively, relative estimates of $\lambda_k(X)$, isn't imposed conditions about priorities of criteria.

Definition 2. *(Definition of a minimum level among all relative estimates of criteria).* The relative level λ in a vector problem represents the lower assessment of a point of $X \in S$ among all relative estimates of $\lambda_k(X)$, $k = \overline{1,K}$:

$$\forall X \in \mathbf{S} \;\; \lambda \le \lambda_k(X),\; k = \overline{1,K}, \qquad (4)$$

the lower level for performance of a condition (4) in an admissible point of $X \in \mathbf{S}$ is defined by a formula

$$\forall X \in \mathbf{S} \;\; \lambda = \min_{k \in K} \lambda_k(X). \qquad (5)$$

Ratios (4) and (5) are interconnected. They serve as transition from operation (5) of definition of min to restrictions (4) and vice versa.

The level λ allows to unite all criteria in a vector problem one numerical characteristic of λ and to make over her certain operations, thereby, carrying out these operations over all criteria measured in relative units. The level λ functionally depends on the $X \in \mathbf{S}$ variable, changing X, we can change the lower level - λ. From here we will formulate the rule of search of the optimum decision.

Definition 3. (*The principle of an optimality with equivalent criteria*).

The vector problem of mathematical programming at equivalent criteria is solved, if the point of $X^o \in S$ and a maximum level of λ^o (the top index o - optimum) among all relative estimates such that is found

$$\lambda^o = \max_{X \in S} \min_{k \in K} \lambda_k(X). \qquad (6)$$

Using interrelation of expressions (4) and (5), we will transform a maximine problem (6) to an extreme problem

$$\lambda^o = \max_{X \in S} \lambda, \qquad (7)$$

$$\lambda \le \lambda_k(X),\; k=\overline{1,K}. \qquad (8)$$

The resulting problem (7)-(8) let's call the λ-problem.

λ-problem (7)-(8) has $(N+1)$ dimension, as a consequence of the result of the solution of λ-problem (7)-(8) represents an optimum vector of $\mathbf{X}^o \in \mathbf{R}^{N+1}$, $(N+1)$ which component an essence of the value of the λ^o, i.e. $\mathbf{X}^o=\{x_1^o, x_2^o, ..., x_N^o, x_{N+1}^o\}$, thus $x_{N+1}^o = \lambda^o$, and $(N+1)$ a component of a vector of $\mathbf{X}^o$ selected in view of its specificity.

The received a pair of $\{\lambda^o, X^o\}=\mathbf{X}^o$ characterizes the optimum solution of λ-problem (7)-(8) and according to vector problem of mathematical programming (1)-(3) with the equivalent criteria, solved on the basis of normalization of criteria and the principle of the guaranteed result. We will call in the optimum solution of $\mathbf{X}^o=\{X^o, \lambda^o\}$, X^o - an optimal point, and λ^o - a maximum level. An important result of the algorithm for solving vector problems (1)-(3) with equivalent criteria is the following theorem.

Theorem 1. (*The theorem of two most contradictory criteria in a vector problem of mathematical programming with equivalent criteria*).

In convex vector problems of mathematical programming at the equivalent criteria which is solved on the basis of normalization of criteria and the principle of the guaranteed result, in an optimum point of $\mathbf{X}^o=\{\lambda^o, X^o\}$ two criteria are always - denote their indexes $q \in \mathbf{K}$, $p \in \mathbf{K}$ (which in a sense are the most contradiction of the criteria $k = \overline{1,K}$), for which equality is carried out:

$$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o),\; q, p \in \mathbf{K},\; X \in S, \qquad (9)$$

and other criteria are defined by inequalities:

$$\lambda^o \le \lambda_k(X^o) \;\; \forall k \in \mathbf{K},\; q \ne p \ne k. \qquad (10)$$

2.3 Mathematical algorithm of the solution of a vector problem with equivalent criteria

For the solution of vector problems of mathematical programming (1)-(3) the methods based on axiomatics of normalization of criteria and the principle of the guaranteed result [4, 15] are offered. Methods follow from an axiom 1 and the principle of an optimality 1.

We will present in the form of a number of steps:

Algorithm of the solution of a vector problem (1)-(3) with equivalent criteria.

Step 1. The problem (1)-(3) by each criterion separately is solved, i.e. for $\forall k \in K_1$ is solved at the maximum, and for $\forall k \in K_2$ is solved at a minimum. As a result of the decision we will receive:

X_k^* - an optimum point by the corresponding criterion, $k=\overline{1,K}$;

$f_k^* = f_k(X_k^*)$ – the criterion size k-th in this point, $k=\overline{1,K}$.

Step 2. We define the worst value of each criterion on $\mathbf{S}$: f_k^0, $k=\overline{1,K}$. For what the problem (1)-(3) for each criterion of $k=\overline{1,K}_1$ on a minimum is solved:

$f_k^0 = \min f_k(X)$, $G(X) \le B$, $X \ge 0$, $k=\overline{1,K}_1$.

The problem (1)-(3) for each criterion on a maximum is solved: f_k^0 = max $f_k(X)$, $G(X) \le B$, $X \ge 0$, $k=\overline{1,K}$ 2.
As a result of the decision we will receive:
$X_k^0=\{x_j, j=\overline{1,N}\}$ - an optimum point by the corresponding criterion, $k=\overline{1,K}$; $f_k^0=f_k(X_k^0)$ – the criterion size k-th a point, X_k^0, $k=\overline{1,K}$.

Step 3. The analysis of a set of points, optimum across Pareto, for this purpose in optimum points of $X^*=\{X_k^*, k=\overline{1,K}\}$ are defined sizes of criterion functions of $F(X^*)=\{f_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\}$ and relative estimates

$$\lambda(X^*) = \{\lambda_q(X_k^*), q=\overline{1,K}, k=\overline{1,K}\},$$

$$\lambda_k(X) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}, \ \forall k \in \boldsymbol{K}:$$

$$F(X^*)=\begin{vmatrix} f_1(X_1^*),\dots, f_k(X_1^*), \\ \dots \\ f_1(X_k^*),\dots, f_k(X_k^*) \end{vmatrix}, \ \lambda(X^*)=\begin{vmatrix} \lambda_1(X_1^*),\dots, \lambda_k(X_1^*), \\ \dots \\ \lambda_1(X_k^*),\dots, \lambda_k(X_k^*) \end{vmatrix}. \quad (11)$$

As a whole on a problem of accordance with o$\forall k\in\boldsymbol{K}$ the relative assessment of $\lambda_k(X)$, $k=\overline{1,K}$ lies within

$$0 \le \lambda_k(X) \le 1, \ \forall k \in \boldsymbol{K}.$$

Discussion. In general on a problem (1)-(3) relative assessment of $\forall k\in\boldsymbol{K}$ lies within $0 \le \lambda_k(X) \le 1$, $\forall k \in \boldsymbol{K}$. Relative estimates of the optimum points $X^*=\{X_k^*, k=\overline{1,K}\}$ (on matrix diagonal $\lambda(X^*)$) are equal to unit: $\lambda_k(X_k^*)=1$, $k=\overline{1,K}$.
It is required to find such point of X^o at which relative estimates of $\lambda_k(X^o)$, $k=\overline{1,K}$, k = were closest to unit.
The step 4 is directed to the solution of this problem.
Step 4. Creation of the λ-problem.
Creation of λ-problem is carried out in two stages: initially built the maximine problem of optimization with the normalized criteria which at the second stage will be transformed to the standard problem of mathematical programming called λ-problem.
For construction maximine a problem of optimization we use definition - relative level $\forall X\in\boldsymbol{S}$ $\lambda=\min_{k\in K}\lambda_k(X)$.
The bottom λ level is maximized on $X\in S$, as a result we will receive a maximine problem of optimization with the normalized criteria.

$$\lambda^o = \max_x \min_k \lambda_k(X), \ G(X) \le B, \ X \ge 0. \quad (12)$$

At the second stage we will transform a problem (12) to a standard problem of mathematical programming:

$$\lambda^o = \max \lambda, \quad \to \quad \lambda^o = \max \lambda, \quad (13)$$

$$\lambda - \lambda_k(X)\le 0, \ k=\overline{1,K}, \ \to \ \lambda - \frac{f_k(X) - f_k^o}{f_k^* - f_k^o} \le 0, \ k=\overline{1,K}, \quad (14)$$

$$G(X) \le B, \ X \ge 0, \quad \to \quad G(X) \le B, \ X \ge 0, \quad (15)$$

where the vector of unknown of X has dimension of N+1: $X=\{\lambda, x_1, \dots, x_N\}$
Step 5. Solution of λ-problem.

λ-problem (13)-(15) is a standard problem of convex programming and for its decision standard methods are used.
As a result of the solution of λ-problem it is received:
$\boldsymbol{X^o}=\{\lambda^o, X^o\}$ - an optimum point;
$f_k(X^o)$, $k=\overline{1,K}$ - values of the criteria in this point;
$\lambda_k(X^o) = \frac{f_k(X^o) - f_k^o}{f_k^* - f_k^o}$, $k=\overline{1,K}$ - sizes of relative estimates;
λ^o - the maximum relative estimates which is the maximum bottom level for all relative estimates of $\lambda_k(X^o)$, or the guaranteed result in relative units, λ^o guarantees that all relative estimates of $\lambda_k(X^o)$ more or are equal λ^o in X^o point to λ^o,

$$\lambda_k(X^o)\ge\lambda^o, \ k=\overline{1,K} \text{ or } \lambda^o \le \lambda_k(X^o), \ k=\overline{1,K}, \ X^o \in S, \quad (16)$$

and according to the theorem the 2 [4, 15] point of $\boldsymbol{X^o}=\{\lambda^o, x_1, \dots, x_N\}$ is optimum across Pareto.

Discussion. In total we have presented in sections 1.1, 1.2, 1.3 "*Methodology of system modeling to base of a problem of vector optimization*" and adoptions of the optimal solution at equivalent criteria. Numerical implementation of methodology of system modeling is presented in sections: economic and technical.

3. ECONOMIC SCIENCE. VECTOR OPTIMIZATION SYSTEM FOR OPTIMAL DECISION MAKING IN ECONOMIC SYSTEM

3.1 A problem of vector optimization in economic systems

In work [19] the analysis of the main economic problems characterizing economic theories of development of firm is carried out. All of them are connected with focus of activity and according to behavior of firm in society. The analysis has shown that further development is connected with the theory of plurality of the purposes which recognizes from the fact that the firm has not one purpose (profit, sales volume, growth), and several (system) is more whole. Developing this direction, in work the numerical model of development of economy of firm to which development pay much attention in Russia and abroad [15 - 20] is constructed.

We will show creation of mathematical model of formation of the annual plan for the enterprises of small and medium business. Economic indicators are considered when forming a vector of variables, criterion and restrictions which are imposed on functioning of firm. The assessment of activity of the enterprise is characterized by a certain set of economic indicators. Focus of such enterprise is reflected in mathematical model in a look by a vector problem of linear programming:

opt $F(X(t))=\{$

$$F_1(X(t))=\{max\ f_k(X(t)) \equiv \sum_{j=1}^{N} c_j^k x_j(t),\ k=\overline{1,K}_1\}, \qquad (17)$$

$$F_2(X(t))=\{min\ f_k(X(t)) \equiv \sum_{j=1}^{N} c_j^k x_j(t),\ \ k=\overline{1,K}_2\}\}, \qquad (18)$$

$$\sum_{j=1}^{N} a_{ij}(t)x_j(t) \le b_i(t),\ i=\overline{1,M}, \qquad (19)$$

$$\sum_{j=1}^{N} c_i a_{ij}(t)x_j(t) \ge b_k(t),\ k\in K,\ x_j(t)\le u_j(t),\ j=\overline{1,N}, \qquad (20)$$

where $F(X(t))$ - vector criterion at which the subset of economic indicators (criteria) of K_1 is required to be maximized, and K_2 - to minimize; $X=\{x_j(t),\ j=\overline{1,N}\}$ – a vector of variables, everyone a component of which defines quantity of j- th of a type of the products included in the plan; c_j^k - economic indicator of k-th of a type of $k=\overline{1,K}_1$, j- th of a type of production characterizing unit which should be received as it is possible (to maximize) above, the sales volume, $c_j^1=p_j^1$, profit of $c_j^2=p_j^2$, $c_j^3=p_j^{dob}$ value added can concern to them. Similarly for criteria of minimization of $k=\overline{1,K}_2$.

For the solution of a vector problem of linear programming (17)-(20) the methods based on normalization of criteria and the principle of the guaranteed result which algorithm of the decision is given.

3.2 The production plan of the enterprise with criteria of maximizing sales, profits, value added

The problem of formation of the production plan of firm of small business is considered. The plan of the enterprise is submitted by two variables x_1, x_2 that allows to show geometrical interpretation of results of the decision.

Creation of model of the production plan of the enterprise it is representable in three stages: the analysis of basic data and statement in the form of a vector problem of linear programming; the solution of VPLP at equivalent criteria; analysis of results, acceptance of a final decision.

3.2.1 Problem definition of the production plan

It is given. The enterprise turns out uniform products of two types, N=2. By production of products one resource (materials) M=1 is used.

We will designate: c_j^1 - market price of production of a unit of production of j-th of a look, $j=\overline{1,2}$, c_j^2 - the profit got by firm from sale of a unit of production of j-th of a look, $j=\overline{1,2}$; a_{ij} – the consumption rate of resources shows what quantity of units of i-th of a resource necessary at production units of production of j-th of a look. In total a_{ij} represent a technological matrix of production which numerical values are presented in tab. 1. In her potential opportunities of the enterprise for everyone of types of the resources b_i, i =1, and also economic indicators are specified: the income c_j^1, profit c_j^2 and a value added c_j^3 from realization of unit of a product of each look.

It is required to define the production plan of the enterprise which includes indicators according to the nomenclature (by types of products) and on volume, i.e. how many products of the corresponding type of a product the enterprise that the income, the profit and a gross value added at their realization was should make as it is possible above, and it is less expense. We have to make mathematical model of a task and solve it.

Table 1 Expenses of resources and operational performance

Look Resources	**Costs of resources of one product**		**Possibilities of firm on resources**
	Look 1	**Look 2**	
Income from a unit of production c_j^1, j=1,2	20	120	To maximize
Resources (kg.) $a_j=\{a_1, a_2\}$	4	5	b=120
Cost of unit of the c1j resource (rub) c_{1j}	2.5	2.5	
Processing of unit of the resource (rub) c_{2j} – (salary)	1.25	21.3	
Prime cost of unit of a resource $c_j=c_{1j}+c_{2j}$ (rub)	3.75	23.8	
Prime cost of a unit of production $p_j=c_j*a_j$ (rub)	15	119	
Profit $c_j^2=c_j^1-p_j$, j=1,2	5	1	To maximize
Value added $c_j^3=c_j^1-c_{1j}$, j=1,2	17.5	117.5	To maximize
Demand for production $u_j=\{u_1, u_2\}$	28	20	
Output	x_1	x_2	To define

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3.2.2 Creation of mathematical model of the production plan

As the unknown we will accept x_1 – the quantity of products of the first look made at the enterprise is similar to x_2 - quantity of units of the second look. For production of such quantity of products it will be required to spend $4x_1 + 5x_2 \le 120$ materials, $x_1 \ge 0$, $x_2 \ge 0$.

The income from realization will make $f_1(X) = 20x_1 + 120x_2$, similar to profit: $f_2(X) \equiv (5x_1 + 1x_2)$ and gross value added of $f_3(X) \equiv (17.5x_1 + 117.5x_2)$.

The purpose of the producer to gain income, profit and a gross value added from production and sale of products as above taking into account restrictions on material resources is possible. From here the target orientation can be expressed by means of a vector problem of linear programming (VPLP):

$$opt\ F(X)=\{max\ f_1(X) \equiv (20x_1 + 120x_2), \quad (21)$$

$$max\ f_2(X) \equiv (5x_1 + 1x_2), \quad (22)$$

$$max\ f_3(X) \equiv (17.5x_1 + 117.5x_2)\}, \quad (23)$$

at restrictions: $4x_1 + 5x_2 \le 120$, (24)

$x_1 \le 28$, $x_2 \le 20$, $x_1 \ge 0$, $x_2 \ge 0$. (25)

Restrictions of the VPLP (21)-(25) are graphically shown in figure 1.

In VPLP the following is formulated: it is required to find the non-negative solution of x_1, x_2, in system of inequalities (24)-(25) it at which the functions $f_1(X)$, $f_2(X)$, $f_3(X)$ accept the greatest possible value.

The linear $f_1(X)$, $f_2(X)$ and $f_3(X)$ functions which maximum is required to be defined together with system of inequalities (24)-(25) form mathematical model of the annual plan of the enterprise (an initial task). For the solution of a vector problem of linear programming (21)-(25) the methods based on normalization of criteria and the principle of the guaranteed result with equivalent criteria and with the set criterion priority presented in [15 -20] are used

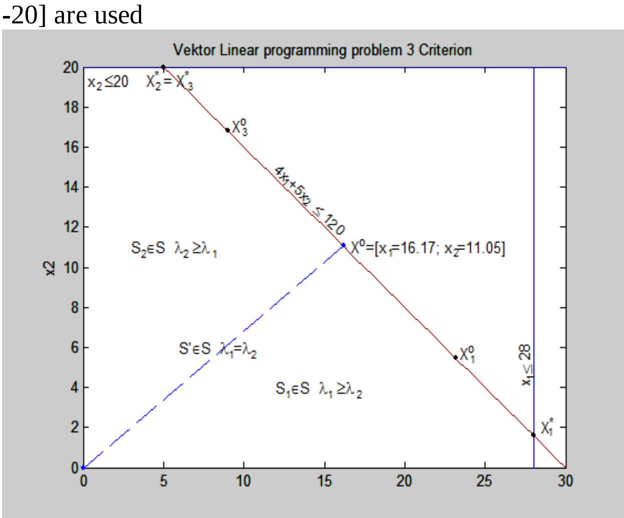

Figure 1. Restrictions, results of the solution of VPLP (21)-(25) [12]

3.2.3 The solution of VPLP at equivalent criteria in the Matlab system

For the solution of a vector problem of linear programming (21)-(25) we will create basic data in the *Matlab* system.

```
cvec = [-20.0 - 120.0;  % Sales volume
        - 5.0 - 1.0;    % profit Volume
        - 17.5 - 117.5] % Volume of a gross value added
a = [4. 5.]             % matrix of linear restrictions
b= [120]                % the vector containing restrictions (bi)
Aeq= []; beq= []        % restriction  like equality
 lb= [0 0]; ub=[28 20]; % a vector of restrictions for variables
```

The algorithm of the solution of VPLP (21)-(25) is represented sequence of steps.

Step 1. Decision on each criterion.

1) The decision on the first criterion (21), (i.e. the model 2a [18] of the maximum sales volume is realized):

[*x1max*,*f1max*]=*linprog*(*cvec*(*1,:*),*a*,*b*,*Aeq*,*beq*,*lb*,*ub*)

where expression in brackets (…) are basic data, expression in square brackets […] results of the decision - *x1max* - a vector of optimum values of variables (an optimum point) by the first criterion; *f1max* – the size of criterion function in this point:

$X_1^* = x1max = \{x_1 = 28.0,\ x_2 = 1.6\}$; $f_1^* = f1max = -141.6$.

2) The decision on the second criterion (22), (i.e. model 1 [18] – receiving the maximum profit is realized):

[*x2max*,*f2max*]=*linprog*(*cvec*(2,:),*a*,*b*,*Aeq*,*beq*,*lb*,*ub*)

$X_2^* = x2max = \{x_1 = 5.0,\ x_2 = 20.0\}$, $f_2^* = f2max = -2500$.

3) The decision on the third criterion (23), (i.e. model 5 [18] - the maximum gross value added - the Japanese model is realized):

[*x3max*,*f3max*]=*linprog*(*cvec*(3,:),*a*,*b*,*Aeq*,*beq*,*lb*,*ub*)

$X_3^* = x3max = \{x_1 = 5.0,\ x_2 = 20.0\}$, $f_3^* = f3max = -2437.5$.

The received points of an optimum of $X_1^* = x1max$, $X_2^* = x2max$, $X_3^* = x3max$ are shown in figure 1, at the same time to $X_2^* = X_3^*$.

Step 2. The worst point of an optimum is determined by each criterion (anti-optimum) by multiplication of criterion by minus unit.

Decision on the first criterion (22):

[*x1min*,*f1min*]=*linprog*(-1**cvec*(*1,:*),*a*,*b*,*Aeq*,*beq*,*lb*,*ub*)

where *x1min* - a vector of optimum values of variables (anti-optimum); *f1min* – the size of criterion function in this point: $X_1^0 = x1min = \{x_1 = 0,\ x_2 = 0\}$; $f_1^0 = f1max = 0$.

Similarly by the second and third criterion:

$X_2^0 = x2min = \{x_1 = 0.0,\ x_2 = 0.0\}$, $f_2^0 = f2min = 0$.

$X_3^0 = x3min = \{x_1 = 0.0,\ x_2 = 0.0\}$, $f_3^0 = f3min = 0$.

Step 3. The system analysis of criteria in VPLP (21)-(25) is made, (i.e. the system of three criteria in optimum points is analyzed). For this purpose in optimum points of X_1^*, X_2^*, X_3^* are defined sizes of criterion functions $F(X^*)=\left\| f_q(X_k^*) \right\|_{k=\overline{1,K}}^{q=\overline{1,K}}$ and relative estimates of $\lambda(X^*)=\left\| \lambda_q(X_k^*) \right\|_{k=\overline{1,K}}^{q=\overline{1,K}}$.

The relative assessment by each criterion is determined by a formula:

$$\lambda_k(X) = \frac{f_k(X) - f_k^o}{f_k^* - f_k^o}, \ \forall k \in \boldsymbol{K}, \qquad (26)$$

where $f_k(X)$ - the criterion size k-th in point $X\in\boldsymbol{S}$; f_k^* - the criterion size k-th in a point of an optimum of $X^*\in\boldsymbol{S}$ received on the first step on $k\in\boldsymbol{K}$ to criterion; f_k^o - the worst size k-th of criterion on an admissible set of $\boldsymbol{S}$ received on the second step. In the *Matlab* system calculation of these functions will be the following:

f=[*cvec*(1,:)**x*1 *cvec*(2,:)**x*1 *cvec*(3,:)**x*1;
cvec(1,:)**x*2 *cvec*(2,:)**x*2 *cvec*(3,:)**x*2;
cvec(1,:)**x*3 *cvec*(2,:)**x*3 *cvec*(3,:)*x3]
d1=-f1max-f1min d2=-f2max-f2min d3=-f3max- f3min
L=[(-*f*(1,1)-*f*1*min*)/*d*1 (-*f*(1,2)-*f*2*min*)/*d*2 (-*f*(1,3)-*f*3*min*)/*d*3;
(-*f*(2,1)- *f*1*min*)/*d*1 (-*f*(2,2)-*f*2*min*)/*d*2 (-*f*(2,3)-*f*3*min*)/*d*3;
(-*f*(3,1)- *f*1*min*)/*d*1 (-*f*(3,2)-*f*2*min*)/*d*2 (-*f*(3,3)-*f*3*min*)/*d*3]

We will receive as a result of the decision:

$$f=\begin{vmatrix} f_1(X_1^*)\ f_2(X_1^*)\ f_3(X_1^*) \\ f_1(X_2^*)\ f_2(X_2^*)\ f_3(X_2^*) \\ f_1(X_3^*)\ f_2(X_3^*)\ f_3(X_3^*) \end{vmatrix} = \begin{vmatrix} 141.6 & 752.0 & 678.0 \\ 45.0 & 2500.0 & 2437.5 \\ 45.0 & 2500.0 & 2437.5 \end{vmatrix},$$

$$L=\begin{vmatrix} \lambda_1(X_1^*)\lambda_2(X_1^*)\lambda_3(X_1^*) \\ \lambda_1(X_2^*)\lambda_2(X_2^*)\lambda_3(X_2^*) \\ \lambda_1(X_3^*)\lambda_2(X_3^*)\lambda_3(X_3^*) \end{vmatrix} = \begin{vmatrix} 1.00 & 0.30 & 0.28 \\ 0.32 & 1.00 & 1.00 \\ 0.32 & 1.00 & 1.00 \end{vmatrix}$$

As a result of the decision from matrix $L=\lambda(X^*)$, we receive that in optimum points of X_1^*, X_2^*, X_3^* criteria of $\lambda_1(X_1^*)=\lambda_2(X_2^*)=\lambda_3(X_3^*)=1$, the others it is less or are equal to unit.

Step 4. λ-problem is under construction:

$$\lambda^o = \max \lambda, \qquad (27)$$

at restrictions $\lambda - \frac{20x_1+120x_2-f_1^0}{f_1^*-f_1^0} \le 0, \qquad (28)$

$$\lambda - \frac{5x_1+1x_2-f_2^0}{f_2^*-f_2^0} \le 0,\ \lambda - \frac{17.5x_1+117.5x_2-f_3^0}{f_3^*-f_3^0} \le 0, \qquad (29)$$

$$4x_1 + 5x_2 \le 120,\ x_1 \le 28,\ x_2 \le 20,\ x_1 \ge 0,\ x_2 \ge 0. \qquad (30)$$

For the solution of a λ-problem (27)-(30) basic data are formed and the appeal to the *linprog* function is presented in the form:

[*Xo, Lo*]=*linprog*(*kL*,*a*0,*b*0,*Aeq*,*beq*,*lbo*,*ubo*).

Results of the solution of λ-problem:
optimum values of variables: X^o={λ=0.6493, x_1=16.175, x_2=11.06};
optimum value of criterion function: λ^o= 0.64693;
The optimum point X^o={x_1=16.1753, x_2=11.0598} is shown on figure 1, we will present her and λ^o in the three-dimensional image in figure 2.

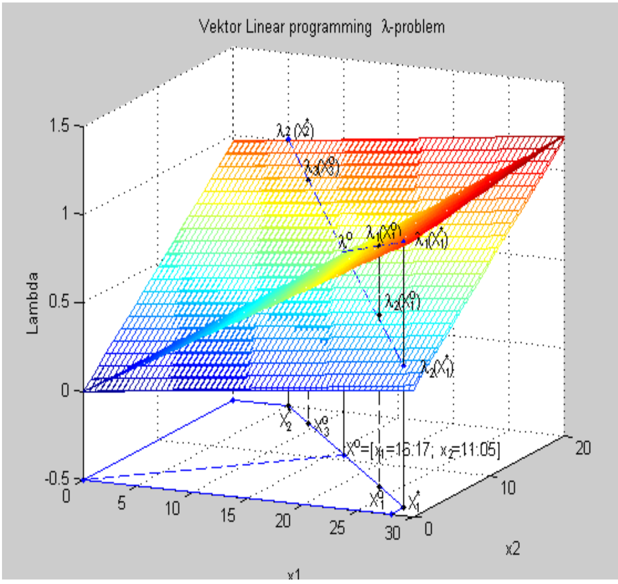

Figure 2. Criteria, results of modeling of the production plan [12]

We will execute check, in an optimum point of X^o we will determine sizes of criterion functions of $F(X^o)=\{f_k(X^o), k=\overline{1,K}\}$, relative estimates of $\lambda(X^o)=\{\lambda_k(X^o), k=\overline{1,K}\}$.

As a result of the decision we will receive:
fXo=[$f_1(X^o)$= 91.9, $f_2(X^o)$=1650.7, $f_3(X^o)$=1582.6],
$\lambda_1(X^o)$= 0.6493, $\lambda_2(X^o)$=0.6603, $\lambda_3(X^o)$=0.6493, i.e. $\lambda^o \le \lambda_k(X^o)$, k=1,2,3.

In figure 2 three normalized $\lambda_1(X)$, $\lambda_2(X)$, $\lambda_3(X)$ planes (the $\lambda_2(X)$, $\lambda_3(X)$ planes coincide) and area of restrictions which general view is shown in figure 1 are presented. In this figure 2 the normalized planes of sales volumes $\lambda_1(X)$ are crossed with profit of $\lambda_2(X)$ and $\lambda_3(X)$. The point of intersection is X^o where $\lambda^o = min(\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o))$ = =0. 6493. Any attempt to increase one of criteria leads to reduction of other criteria, i.e. the point of X^o is optimum across Pareto, and λ^o – the guaranteed result in relative units.

4. TECHNICAL SCIENCE. METHODOLOGY OF MODELING OF TECHNICAL SYSTEMS IN THE CONDITIONS OF DEFINITENESS AND UNCERTAINTY

4.1 Technology of research, creation of mathematical model of technical systems and decision making

The methodology of process of construction of the technical systems (TS) mathematical model and decision-making on its basis is intended for the analysis and TS synthesis at a design stage and operation. The main attention is paid, first, to creation of the TS model and methods of the solution of VPMP, secondly, a place of these models and methods in problems of design of technical (engineering) systems. The block diagram of methodology is submitted in figure 3 and described, how sequence of a number of steps (blocks), [5, 9].

Block 0. The specification on developed products where the purposes and requirements to technical systems are formulated is formed.

Block 1. For research of the physical processes proceeding in the TS, and creation of mathematical models of such processes fundamental laws of physics are used: modeling of magnetic, temperature fields; conservation laws of energy, movement, etc.

At the same stage for the solution of the problems which are cornerstone of studied physical processes, numerical methods are developed. As a rule, modeling and calculations are carried out by means of the software developed for this purpose and computer facilities. Then the software is tested for adequacy to real physical data.
If the main physical processes proceeding in the TS are known, and functional dependence of each characteristic and restrictions on the TS parameters is known further, such situation is called "modeling in the conditions of definiteness". If physical processes in the TS are insufficiently studied, such situation is called "modeling in the conditions of uncertainty". In this case, construction of experimental (regression) models associated with the analysis of input and output data [8].

Block 2. The full list of all functional characteristics of technical systems and parameters on which these characteristics depend is formed. Their verbal description is given.

Block 3. The technical and information interrelation of all TS components is established, i.e. the structure is under construction. Here the problem of a choice of the best (in any sense) TS structures is solved, i.e. the problem of structural optimization [6] is carried out.

Block 4. TS mathematical model formation.
It includes four stages.
4.1. Definition of the purposes and indicators of functioning of the TS.
Quality of functioning of the TS by any is defined by a set of technical (output) characteristics which represent a quantitative measure of reflection of requirements to TS properties.

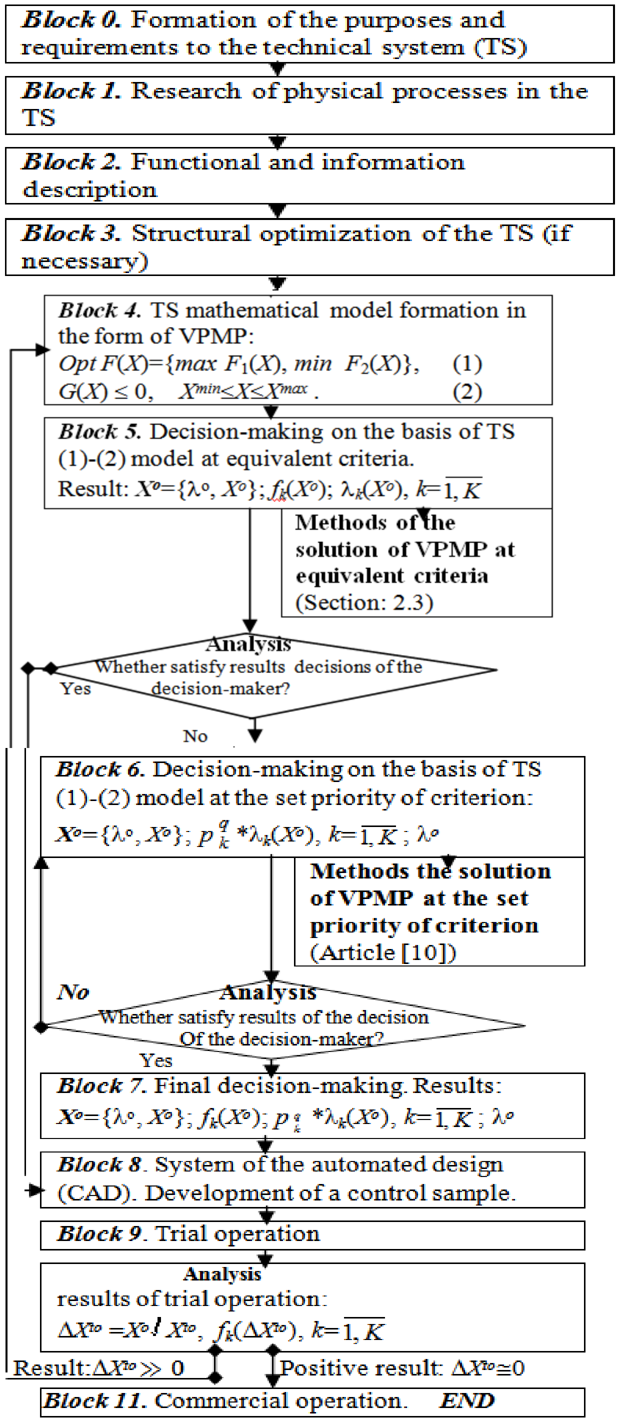

Figure 3. The block diagram of process of design and TS mathematical model place in decision-making [5, 9].

For electronic schemes such characteristics are: output power, speed, accuracy assessment, dimensions, etc. For engines - the output power, speed, efficiency, etc. We will designate set of all vector characteristics a set "***K***", and an index $k = \overline{1, K}$.

4.2. Identification of a vector of the TS variables.
The technical system in a statics is investigated. Major factors and parameters which remain constants for the studied period of time are identified. The variable parameters which size it is desirable to determine and which size can change in the course of design are identified. Them also call operated parameters or design parameters. They are in turn subdivided into internal and external parameters. For electronic schemes internal parameters are: resistance, capacities, inductance, coefficients of strengthening of transistors, etc. For mechanical systems, for example, engines: combustion chamber volume, piston stroke, their quantity, etc. External parameters are the factors connected with environment (for example, temperature), power supplies, etc.

These parameters usually also are subject to definition.
We will designate a vector of design data (a vector of variables) the TS through $X = \{x_j, j = \overline{1, N}\}$, where ***N*** - a set of indexes (*N*- their number) variables. Limits of change of a vector of variables come to light:
$x_j^{\min} \leq x_j \leq x_j^{\max}, \quad j=\overline{1, N}$, or $X^{min} \leq X \leq X^{max}$,
where $x_j^{\min}$, $x_j^{\max}$, $\forall j \in \mathbf{N}$, bottom and top limits of change of a vector of variables. These relations are called parametric restrictions.

Note. Blocks 0, 1,...,4.2 formed by the designer of a technical system (from any industry). For example, from the electro - technical industry is the designer of the motor.

4.3 . Formation of a vector of criteria of the TS.
The criterion is a measure of a quantitative estimation of functional dependence of a vector of variables *X* from each output characteristic of $k=\overline{1, K}$. The set of functional dependences of characteristics represents vector criterion:

$$F(X) = \{f_k(X), k = \overline{1, K}\},$$

where ***K****(K)* is the set of (*K*- number) indexes of criteria of the TS.
Optimization on one of characteristics (criterion) led to deterioration of other characteristics of the TS, as a result the chosen design decision was insolvent. This circumstance also constrained wide use of methods of optimization in the analysis and TS synthesis. The solution of a question, in our opinion, is consolidated to creation of mathematical model which would be adequate to the TS, i.e. would consider all characteristics of the TS at its functioning at the same time.

4.4. The definition of functional dependencies between constraints and parameters of the TS. Are investigated and imposed on functioning of the TS of restriction of four types: the restrictions which are put forward by the specification on creation of the TS; technological restrictions; the restrictions connected with physical processes, proceeding in the TS; restrictions on functioning the TS connected with environment.

Functional dependence of parameters among themselves, and, according to technical requirements to entrance and output parameters is established:

$X^{min} \leq X \leq X^{max}$ - parametrical restrictions.

Taking into account the admissible range of change of variables of restriction in a symbolical form it is possible to present in the form of inequalities:

$$G(X) \leq 0 \text{ or } (g_1(X) \leq 0 \;\; g_2(X) \leq 0 \ldots g_M(X) \leq 0)^{\mathrm{T}},$$

where ***M****(M)* - a set (number) of restrictions of the TS.
4.5. Let part of characteristics of $f_k(X)$, $k = \overline{1, K}_1$, $\mathbf{K_1} \subset \mathbf{K}$, on quantity it is desirable to receive as much as possible (i.e. the corresponding criteria are maximized), and part of $f_k(X)$, $k=\overline{1, K}_2$, $\mathbf{K_2} \subset \mathbf{K}$ are minimized. Taking into account these requirements the mathematical model solving as a whole a problem of a choice of the optimum design decision (a choice of the TS optimum parameters), it is possible to present in the form of a vector problem of mathematical programming. Figure 3, the block 4.

We assume that the VPMP belongs to the class of convex problems, and the set of admissible points of ***S*** presented by restrictions isn't empty and represents a compact. From here it is possible to determine an optimum by any of criteria "***K***". VPMP is the TS model in a statics, but such model can be used for research of dynamic processes for the small period of time [5].

Block 5. Decision-making on the basis of TS model at equivalent criteria.
The made decision is defined by the solution of a vector problem at equivalent criteria, i.e. lack of a priority on any criterion.

Methods of the decision are based on normalization of criteria and the principle of the guaranteed result are presented in section 2.3. Result of the decision:
$X^o = \{\lambda^o, \; X^o\}$; $f_k(X^o)$, $k=\overline{1, K}$; $\lambda_k(X^o)$, $k=\overline{1, K}$; λ^o. (Designations in section 2.3).

The analysis of design parameters of X^o and characteristics of $f_k(X^o)$, $k=\overline{1,K}$ technical system is made. If they meet requirements of the decision-maker, we go to block 8, otherwise the next block.

Block 6. Decision-making on the basis of TS model at the set priority of criterion.
The decision received in the previous block is defined proceeding from equivalence of criteria of the TS. In actual practice the priority (preference) of TS any of criteria, for example, $q\in\boldsymbol{K}$ is usually imposed. The decision in this case gets out of a set of points of $\boldsymbol{S_q}\in\boldsymbol{S}$, lying between points of X^o and X_q^*, $q\in\boldsymbol{K}$.

Methods of the decision are based on normalization of criteria, the principle of the guaranteed result and axiomatics of a priority of criterion in VPMP [4, 10, 15].
The analysis of design data of X^o and characteristics of $f_k(X^o)$, $k=\overline{1,K}$ technical system with a criterion priority is made. If they meet requirements of the decision-maker, the previous block is passed to the block 7, differently.

Block 7. Final decision-making at the set priority of criterion. Results: $\boldsymbol{X^o}=\{\lambda^o, X^o\}$; $f_k(X^o)$, $p_k^q*\lambda_k(X^o)$, $k=\overline{1,K}$; λ^o determine parameters and characteristics of technical system.
Block 8. Computer-aided design (CAD).
On the basis of design parameters of X^o by means of system of the automated design project documentation of the TS which functioning is defined by characteristics of $f_k(X^o)$, $k=\overline{1,K}$ is formed. Built a prototype that is passed into pilot operation.
Block 9. Trial operation (*to*).

Block 10. By results of trial operation experimental data of design data X^{to} and functional characteristics of $f_k(X^{to})$, $k=\overline{1,K}$ can be obtained. They are compared to the X^o parameters from mathematical model
$\Delta X^{to}=X^o - X^{to}$, $f_k(\Delta X^{to})$, $k=\overline{1,K}$.
If deviations of are close to zero: $\Delta X^{to}\cong 0$, TS is put into commercial operation. If deviations are considerable: $\Delta X^{to}\gg 0$, by results of trial operation the second stage of improvement of model begins.

Block 11. Commercial operation.
Applied part of modeling of the TS it is representable in the form of methodology of the solution of VPMP and we will show on test examples of the TS models realized in Matlab system.

4.2 Creation of mathematical model of technical system

The problem of a choice of optimum parameters of technical systems according to functional characteristics arises during the studying, the analysis and design of technical systems and is connected with quality production.

The problem includes the solution of the following tasks:
Creation of mathematical model which defines interrelation of each functional characteristic from parameters of technical system i.e. is formed of the vector problem of mathematical programming;

Choice of methods of the decision: we suggest using the methods based on normalization of criteria and the principle of the guaranteed result with equivalent criteria;
The software which realizes these methods is developed [9-14].

The technical system which functioning depends on $\boldsymbol{N}$ - a set of design data is considered[1]: $X=\{x_1\ x_2\ \dots\ x_N\}$, N - number of parameters, each of which lies in the set limits

$$x_j^{min}\le x_j\le x_j^{max},\ j=\overline{1,N},\ \text{or}\ X^{min}\le X\le X^{max}, \qquad (31)$$

where x_j^{min}, x_j^{max}, $\forall j\in\boldsymbol{N}$ - lower and top limits of change of a vector of parameters of technical system.
The result of functioning of technical system is defined by a set $\boldsymbol{K}$ to technical characteristics of $f_k(X)$, $k=\overline{1,K}$ which functionally depend on design data $X=\{x_j,\ j=\overline{1,N}\}$, in total they represent a vector function:

$$F(X) = (f_1(X)\ \ f_2(X)\ \dots\ f_K(X))^T. \qquad (32)$$

The set of characteristics (criteria) to is subdivided into two subsets $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$: $\boldsymbol{K}=\boldsymbol{K}_1\cup\boldsymbol{K}_2$
$\boldsymbol{K}_1$ is a subset of technical characteristics which numerical sizes it is desirable to receive as it is possible above:

$$f_k(X)\ \rightarrow \mathbf{max},\ k=\overline{1,K_1}.$$

$\boldsymbol{K}_2$ - it subsets of technical characteristics which numerical sizes it is desirable to receive as it is possible below: $f_k(X)\rightarrow\mathbf{min}$, $k=\overline{K_1+1,K}$, $\boldsymbol{K}_2\equiv\overline{K_1+1,K}$.

Mathematical model of technical system which solves in general a problem of a choice of the optimum design decision (a choice of optimum parameters), we will present in the form of a vector problem of mathematical programming.

$Opt\ F(X) = \{\mathbf{max}\ F_1(X) = \{\mathbf{max}\ f_k(X), k = \overline{1, K_1}\},$ (33)

$\mathbf{min}\ F_2(X) = \{\mathbf{min}\ f_k(X), k = \overline{1, K_2}\}\},$ (34)

$G(X) \le 0,$ (35)

$x_j^{\min} \le x_j \le x_j^{\max}, j = \overline{1, N},$ (36)

where X - a vector of operated variable (design data) from (1);

$F(X)=\{f_k(X), k=\overline{1,K}\}$ - criterion which everyone a component submits the characteristic of technical system (32) which is functionally depending on a vector of variables X;

in (35) $G(X)=(g_1(X)\ g_2(X) \dots g_M(X))^{\mathrm{T}}$ – vector function of the restrictions imposed on functioning of technical system, $\boldsymbol{M}$ – a set of restrictions.

Restrictions are defined proceeding in them technological, physical and to that similar processes and can be presented by functional restrictions, for example,

$$f_k^{\min} \le f_k(X) \le f_k^{\max}, k=\overline{1,K}.$$

It is supposed that the $f_k(X)$, $k=\overline{1,K}$ functions are differentiated and convex, $g_i(X)$, $i=\overline{1,M}$ are continuous, and (35)-(36) set of admissible points of $\boldsymbol{S}$ set by restrictions isn't empty and represents a compact:

$$\boldsymbol{S}=\{X \in \boldsymbol{R}^N \mid G(X) \le 0, X^{min} \le X \le X^{max}\} \ne \emptyset.$$

Criteria and restrictions (33)-(36) form mathematical model of technical system. It is required to find such vector of the $X^o \in \boldsymbol{S}$ parameters at which everyone a component the vector - functions $F_1(X)=\{f_k(X), k=\overline{1,K_1}\}$ accepts the greatest possible value, and a vector - functions $F_2(X)=\{f_k(X), k=\overline{1,K_2}\}$ are accepted by the minimum value.

To a substantial class of technical systems which can be presented by a vector task (33)-(36), it is possible to refer their rather large number of tasks from various branches of economy of the state: electrotechnical, aerospace, metallurgical (choice of optimal structure of material), etc. In this article for technical system are considered in a statics. But technical systems can be considered in dynamics, using differential-difference methods of transformation [5], conducting research for a small discrete period $\Delta t \in T$.

4.3 The mathematical model of technical system in the conditions of definiteness and uncertainty in total

Conditions of definiteness are characterized by that functional dependence of each characteristic and restrictions on parameters of technical system [6, 8, 13] is known.

Conditions of uncertainty are characterized by that there is no sufficient information on functional dependence of each characteristic and restrictions from parameters [6, 9, 14, 16].

In real life of a condition of definiteness and uncertainty are combined. The model of technical system also has to reflect these conditions. We will present model of technical system in the conditions of definiteness and uncertainty in total [14]:

$Opt\ F(X) = \{\mathbf{max}\ F_1(X) = \{\mathbf{max}\ f_k(X), k = \overline{1, K_1^{def}}\}, \mathbf{max}\ I_1(X) \equiv \{\mathbf{max}\{f_k(X_i, i=\overline{1,M})\}^{\mathrm{T}}, k=\overline{1,K_1^{unc}}\},$ (37)

$\mathbf{min}\ F_2(X)=\{\mathbf{min}\ f_k(X), k=\overline{1,K_2^{def}}\}, \mathbf{min}\ I_2(X) \equiv \{\mathbf{min}\{f_k(X_i, i=\overline{1,M})\}^{\mathrm{T}}, k=\overline{1,K_2^{unc}}\}\},$ (38)

at restrictions

$f_k^{\min} \le f_k(X) \le f_k^{\max}, k=\overline{1,K},$ (39)

$x_j^{\min} \le x_j \le x_j^{\max}, j = \overline{1, N},$ (40)

where X - a vector of operated variable (design data) equivalent (31); $F(X)=\{F_1(X)\ F_2(X)\ I_1(X), I_2(X)\}$ - vector criterion which everyone a component represents a vector of criteria (characteristics) of technical system (32) which functionally depend on discrete values of a vector of variables X where K_1^{def}, K_2^{def} (*definiteness*), K_1^{unc}, K_2^{unc} (*uncertainty*) the set of criteria of *max* and *min* created in the conditions of definiteness and definiteness; in (39) $f_k^{\min} \le f_k(X) \le f_k^{\max}$, $k=\overline{1,K}$ – a vector function of the restrictions imposed on functioning of technical system $x_j^{\min} \le x_j \le x_j^{\max}, j = \overline{1, N}$ – parametrical restrictions.

Elimination of uncertainty consists in use of qualitative and quantitative descriptions of technical system which can be received, for example, by the principle "entrance exit". Transformation of basic data "entrance exit" to functional dependence is carried out by use of mathematical methods (the regression analysis) [8, 14].

4.4 Numerical problem of modeling of technical system

We will consider a task "Numerical modeling of technical system" in which data on some set of functional characteristics (definiteness conditions), discrete values of characteristics (an uncertainty condition) and the restrictions imposed on functioning of technical system are known. The numerical problem of modeling of technical system is considered with equivalent criteria and with the set criterion priority.

It is given. The technical system, which functioning is defined by two parameters $X=\{x_1, x_2\}$ – a vector (operated) variables. Basic data for the solution of a task are five characteristics (criterion) of

$F(X)=\{f_1(X), f_2(X), f_3(X), f_4(X), f_5(X)\}$, which size of an assessment depends on a vector of X. For characteristics of $f_1(X)$, $f_2(X)$, $f_5(X)$ functional dependence on parameters X (a definiteness condition) is known:

$$f_1(X)= 67.425+ 0.02225*x_1+ 0.00239*x_1^2- 0.05625*x_2+ 0.00029*x_2^2+ 0.0021232*x_1*x_2,$$
$$f_2(X)= 4456.3 - 2.315*x_1 + 0.239*x_1^2 + 2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2,$$
$$f_5(X) = 281.7 - 0.7*x_1+ 0.01*x_1^2 + 0.36*x_2 - 0.019*x_2^2 +0.022*x_1*x_2. \quad (41)$$

Functional restrictions:

$$1000 \le f_2(X) \equiv 4456.3 - 2.315*x_1+0.239*x_1^2+2.805*x_2 - 0.037*x_2^2-0.22192*x_1*x_2 \le 3100. \quad (42)$$

Parametrical restrictions:

$$25\le x_1\le 100,\ 25\le x_2\le 100. \quad (43)$$

For the third and fourth characteristic results of experimental data are known: sizes of parameters and corresponding characteristics (uncertainty condition). Numerical values of parameters X and characteristics of $y_3(X)$, $y_4(X)$ are presented in table 2.

Table 2. Numerical values of parameters and characteristics of technical system

x_1	x_2	$y_3(X)$→**max**	$y_4(X)$→**min**
25	25	1148	490.9
25	50	1473	483.1
25	75	1798	557.3
25	100	2122	521.5
50	25	725	498.1
50	50	968	521.5
50	75	1212	549.9
50	100	1456	578.3
75	25	440	507.3
75	50	572	549.9
75	75	734	592.5
75	100	897	635.1
100	25	202	521.5
100	50	284	578.3
100	75	385	635.1
100	100	446	691.9

In the made decision, assessment size of the first, second and the third characteristic (criterion) is possible to receive above (max), for the fourth and five characteristic is possible below (min). Parameters $X=\{x_1, x_2\}$ change in the following limits: $x_1, x_2 \in [25.\ 50.\ 75.\ 100]$.
It is required. To make the best decision (optimum).

Methodology of modeling of technical system in the conditions of definiteness and uncertainty.

1). Creation of mathematical model of technical system.

1.1. ***Construction in the conditions of definiteness*** is defined by functional dependence of each characteristic and restrictions on parameters of technical system. In our example two characteristics (35) and restrictions (36)-(37) are known:

$$f_1(X) = 67.425+ 0.02225*x_1+ 0.00239*x_1^2- 0.05625*x_2+ 0.00029*x_2^2+ 0.0021232*x_1*x_2,$$
$$f_2(X) = 4456.3 - 2.315*x_1 + 0.239*x_1^2 + 2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2,$$
$$f_5(X) = 281.7 - 0.7*x_1+ 0.01*x_1^2 + 0.36*x_2 - 0.019*x_2^2 +0.022*x_1*x_2. \quad (44)$$

Functional restrictions:

$$3800 \le f_2(X) \equiv 4456.3 - 2.315*x_1 + 0.239*x_1^2 + 2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2 \le 5500. \quad (45)$$

Parametrical restrictions:

$$25\le x_1\le 100,\ 25\le x_2\le 100. \quad (46)$$

These data are used further at creation of mathematical model of technical system.

1.2. Construction in the conditions of uncertainty consists in use of the qualitative and quantitative descriptions of technical system received by the principle "entrance exit" in table 1. Transformation of information (basic data of $y_3(X)$, $y_4(X)$) to a functional type of $f_3(X)$, $f_4(X)$ is carried out by use of mathematical methods (the regression analysis) [8, 13]. As a result have received the functions $f_3(X)$, $f_4(X)$ presented in (49), (50).

1.3. Creation of mathematical model of technical system (The general part for conditions of definiteness and uncertainty).
For creation of mathematical model of technical system we used: the functions received conditions of definiteness (44) and uncertainty (49), (50); functional restrictions (45); parametrical restrictions (46).

We considered functions as the criteria defining focus of functioning of technical system. A set of criteria $\boldsymbol{K}$=5 included three criteria of $f_1(X), f_2(X), f_3(X)$ →max and two $f_4(X), f_5(X)$ →min. As a result model of functioning of technical system was presented a vector problem of mathematical programming:

$$opt\ F(X)=\{max$$

$$F_1(X)=\{max f_1(X)\equiv 67.425+0.02225*x_1+0.00239*x_1^2-0.05625*x_2+0.00029*x_2^2+0.0021232*x_1*x_2, \quad (47)$$

$$max\ f_2(X)\equiv 4456.3 - 2.315*x_1 + 0.239*x_1^2 + 2.805*x_2 - 0.037*x_2^2 - 0.22192*x_1*x_2, \quad (48)$$

$$max\ f_3(X)\equiv 1273.5-19.919*x_1+ 0.0854*x_1^2 +16.071*x_2+ 0.001*x_2^2-0.13034*x_1*x_2\}, \quad (49)$$

$$min\ F_2(X)=\{min\ f_4(X)\equiv 481.7 - 0.6915*x_1+ 0.0047*x_1^2 + 0.3535*x_2 - 0.0023*x_2^2 +0.021808*x_1*x_2, \quad (50)$$

$$min\ f_5(X) = 281.7 - 0.7*x_1+ 0.01*x_1^2 + 0.36*x_2 - 0.019*x_2^2 +0.022*x_1*x_2\}\}, \quad (51)$$

at restrictions

$$3800 \le f_2(X)\equiv 4456.3 - 2.315*x_1+0.239*x_1^2+ 2.805*x_2-0.037*x_2^2 -0.22192*x_1*x_2 \le 5500 \quad (52)$$

$$25\le x_1\le 100,\ 25\le x_2\le 100. \quad (53)$$

The vector problem of mathematical programming represents model of adoption of the optimum decision in the conditions of definiteness and uncertainty in total.

2). The solution of a vector problem of mathematical programming - model of technical system with equivalent criteria and with the set criterion priority.
***2.1. The solution of a vector problem (47)-(53) with equivalent criteria* was submitted as sequence of steps.**

Step 1. Problems (47)-(53) were solved by each criterion separately, thus used the function *fmincon* (…) of *Matlab* system [14], the appeal to the function *fmincon* (…) is considered in [8, 13]. As a result of calculation for each criterion we received optimum points: X_k^* and $f_k^* = f_k(X_k^*)$, $k=\overline{1,K}$ – sizes of criteria in this point, i.e. the best decision on each criterion: $X_1^*=\{x_1=100,\ x_2=100\}$, $f_1^* = f_1(X_1^*) = -112.06$;

$X_2^*=\{x_1=97.16,\ x_2=48.09\}$, $f_2^* = f_2(X_2^*) = -5500$;

$X_3^*=\{x_1=25,\ x_2=100\}$, $f_3^* = f_3(X_3^*) = -2120.15$;

$X_4^* = \{x_1=25,\ x_2=25\}$, $f_4^* = f_4(X_4^*) = 488.38$;

$X_5^*=\{x_1=25,\ x_2=68.41\}$, $f_4^* = f_4(X_4^*) = 243.78$.

Restrictions (53) and points of an optimum $X_1^*,\dots, X_5^*$ in coordinates $\{x_1, x_2\}$ are presented on figure 4.

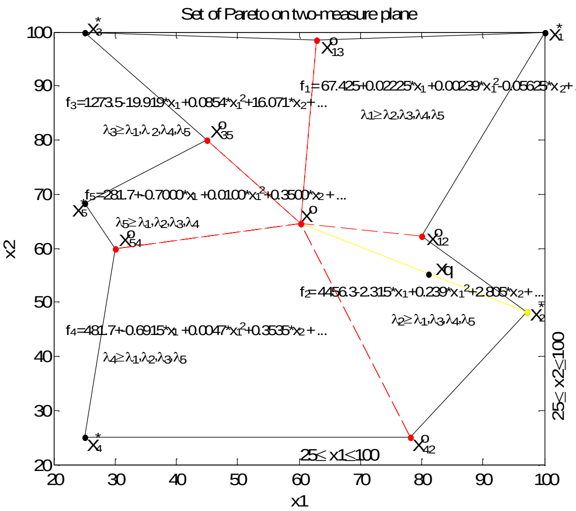

Figure 4. Pareto's great number, $S^o\subset S$ in two-dimensional system of coordinates

Step 2. We defined the worst unchangeable part of each criterion (anti-optimum): $X_1^0=\{x=25,\ x_2=25\}$, $f_1^0=f_1(X_1^0)=69.57$; $X_2^0=\{x_1=37.32,\ x_2=98.88\}$, $f_2^0=f_2(X_2^0)=3800$; $X_3^0=\{x_1= 83.1,\ x_2=25\}$, $f_3^0=f_3(X_3^0)=339.7$; $X_4^0=\{x_1=100,\ x_2=100\}$, $f_4^0 = f_2(X_4^0)=-689.9$. (Top index zero).

Step 3. We made the analysis of a set of points, optimum across Pareto. In points of an optimum of $X^*=\{X_1^*, X_2^*, X_3^*, X_4, X_5^{**}\}$ sizes of criterion functions of $F(X^*)=\left\| f_q(X_k^*)\right\|_{q=\overline{1,K}}^{k=\overline{1,K}}$ determined. Calculated a vector of $D=(d_1\ \ d_2\ \ d_3\ d_4\ d_5)^T$ - deviations by each criterion on an admissible set of $\mathbf{S}$: $d_k = f_k^*-f_k^0$, $k=\overline{1,5}$, and matrix of relative estimates of

$$\lambda(X^*)=\left\|\lambda_q(X_k^*)\right\|_{q=\overline{1,K}}^{k=\overline{1,K}}, \text{ where } \lambda_k(X)=(f_k^*-f_k^0)/d_k.$$

$$F(X^*)=\begin{vmatrix} -112.1 & -4306.1 & -449.3 & 690.0 & 377.7 \\ -100.0 & -5500.0 & -310.5 & 572.5 & 384.3 \\ -72.1 & -3903.5 & -2120.2 & 534.2 & 171,4 \\ -69.6 & -4456.1 & -1149.8 & 488.4 & 281.3 \\ -70.6 & -4187.0 & 1710.1 & 518.1 & 243.8 \end{vmatrix},\quad D=\begin{vmatrix} 42.48 \\ 1700.0 \\ 1780.5 \\ -201.6 \\ -154.2 \end{vmatrix},$$

$$\lambda(X^*)=\begin{vmatrix} 1.0000 & 0.2977 & 0.0616 & 0 & 0.1312 \\ 0.7170 & 1.0000 & -0.0164 & 0.5829 & 0.0887 \\ 0.0584 & 0.0609 & 1.0000 & 0.7726 & 1.4692 \\ 0 & 0.3859 & 0.4550 & 1.0000 & 0.7564 \\ 0.0244 & 0.2276 & 0.7697 & 0.8527 & 1.0000 \end{vmatrix}$$

Step 4. Creation of λ-problem is carried out in two stages: originally the maximine problem of optimization with the normalized criteria is under construction:

$$\lambda^o = \max_x \min_k \lambda_k(X),\ G(X)\le 0,\ X \ge 0,$$

which at the second stage was transformed to a standard problem of mathematical programming (λ-problem):

$$\lambda^o = max\ \lambda, \qquad (54)$$

at restrictions

$$\lambda - \frac{(67.4+0.02225*x_1+0.0024*x_1^2-0.05625*x_2+0.00029*x_2^2+0.002123*x_1*x_2-f_1^o)}{f_1^*-f_1^o} \le 0, \qquad (55)$$

$$\lambda - \frac{(4456.3-2.315*x_1+0.239*x_1^2+2.605*x_2-0.037*x_2^2-0.222*x_1*x_2-f_2^o)}{f_2^*-f_2^o} \le 0, \qquad (56)$$

$$\lambda - \frac{(1273.5-19.92*x_1+0.0854*x_1^2+16.07*x_2+0.01*x_2^2-0.1303*x_1*x_2-f_3^o)}{f_3^*-f_3^o} \le 0, \qquad (57)$$

$$\lambda - \frac{(481.7-0.6915*x_1+0.0047*x_1^2+0.3535*x_2-0.0023*x_2^2+0.0218*x_1*x_2-f_4^o)}{f_4^*-f_4^o} \le 0, \qquad (58)$$

$$\lambda - \frac{(281.7-0.7*x_1+0.01*x_1^2+0.36*x_2-0.019*x_2^2+0.022*x_1*x_2-f_5^o)}{f_5^*-f_5^o} \le 0, \qquad (59)$$

$$3800 \le f_2(x) \le 5500;\ 0 \le \lambda \le 1,\ 25 \le x_1 \le 100,\ 25 \le x_2 \le 100, \qquad (60)$$

where the vector of unknown had dimension of N+1: $\boldsymbol{X}=\{x_1, \ldots, x_N, \lambda\}$. Appeal to function fmincon(), [15]: [Xo,Lo]=fmincon('Z_TehnSist_4Krit_L',X0,Ao,bo,Aeq, beq,lbo,ubo, 'Z_TehnSist_LConst',options).

As a result of the solution of a vector problem of mathematical programming (61)-(66) at equivalent criteria and λ-problem corresponding to it (54)-(55) received:

$\boldsymbol{X^o}=\{X^o, \lambda^o\}=\{X^o=\{x_1=60.36,\ x_2=64.52,\ \lambda^o=0.3236\}$ - an optimum point – design data of technical system, point $\boldsymbol{X^o}$ is presented in figure 1;

$f_k(X^o)$, $k=\overline{1,K}$ - sizes of criteria (characteristics of technical system):

$$\{f_1(X^o)=83.3,\ f_2(X^o)=4350.1,\ f_3(X^o)=915.8,\ f_4(X^o)=555.2,\ f_5(X^o)=305.7\}; \qquad (61)$$

$\lambda_k(X^o)$, $k=\overline{1,K}$ - sizes of relative estimates:

$$\{\lambda_1(X^o)=0.3236,\ \lambda_2(X^o)=0.3236,\ \lambda_3(X^o)=0.3236,\ \lambda_4(X^o)=0.6683,\ \lambda_5(X^o)=0.5984\}; \qquad (62)$$

λ^o=0.3236 is the maximum lower level among all relative estimates measured in relative units:

$\lambda^o=min\ (\lambda_1(X^o), \lambda_2(X^o), \lambda_3(X^o), \lambda_4(X^o), \lambda_5(X^o))=0.3236$.

A relative assessment - λ^o call the guaranteed result in relative units, i.e. $\lambda_k(X^o)$ and according to the characteristic of technical $f_k(X^o)$ system it is impossible to improve, without worsening thus other characteristics.

We will notice that according to the theorem 1, in $\boldsymbol{X^o}$ point criteria 1, 2, 3 are contradictory. This contradiction is defined by equality of $\lambda_1(X^o)=\lambda_2(X^o)=\lambda_3(X^o)=\lambda^o=0.3236$, and other criteria an inequality of $\{\lambda_4(X^o)=0.6683, \lambda_5(X^o)=0.5984\}>\lambda^o$.

Thus, the theorem 1 forms a basis for determination of correctness of the solution of a vector task. In a vector problem of mathematical programming, as a rule, for two criteria equality is carried out:

$\lambda^o = \lambda_q(X^o) = \lambda_p(X^o)$, $q, p \in \boldsymbol{K}$, $X \in S$, (in our example of such criteria three)

and for other criteria is defined as an inequality:

$$\lambda^o \le \lambda_k(X^o)\ \ \forall k \in \boldsymbol{K},\ q \ne p \ne k.$$

In an admissible set of points of $\boldsymbol{S}$ formed by restrictions (60), optimum points X_1^*, X_2^*, X_3^*, X_4^*, X_5^* united in a contour, presented a set of points, optimum across Pareto, to $\boldsymbol{S^o} \subset \boldsymbol{S}$. For specification of border of a great number of Pareto calculated additional points: X_{12}^o, X_{13}^o, X_{35}^o, X_{54}^o, X_{42}^o which lie between the corresponding criteria.

Points: X_{12}^o, X_{13}^o, X_{35}^o, X_{54}^o, X_{42}^o are presented in figure 4. Coordinates of these points, and also characteristics of technical system in relative units of $\lambda_1(X)$, $\lambda_2(X)$, $\lambda_3(X)$, $\lambda_4(X)$, $\lambda_5(X)$ are shown in figure 5 in three measured space, where the third axis of λ - a relative assessment.

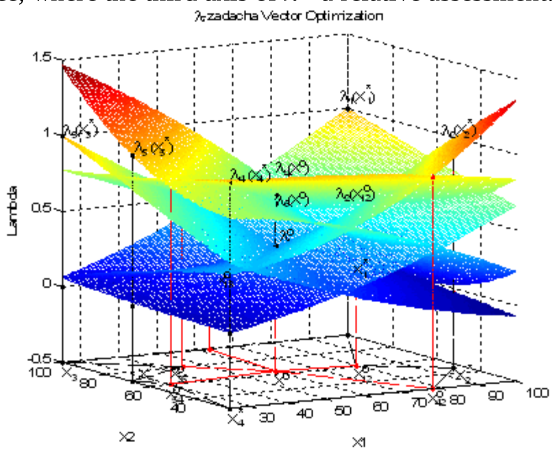

Figure 5. The solution of λ-problem in three-dimensional system of coordinates of x_1, x_2 and λ.

Collectively, the submitted version:

- point of the optimum - $\boldsymbol{X^o}=\{X^o, \lambda^o\}=\{X^o=\{x_1, x_2\}, \lambda^o\}$;
- characteristics of $f_1(X^o)$, $f_2(X^o)$, $f_3(X^o)$, $f_4(X^o)$, $f_5(X^o)$;
- relative estimates of $\lambda_1(X^o)$, $\lambda_2(X^o)$, $\lambda_3(X^o)$, $\lambda_4(X^o)$, $\lambda_5(X^o)$;
- maximum λ^o relative level such that $\lambda^o \le \lambda_k(X^o)$ $\forall k \in \boldsymbol{K}$ - there is *an optimum decision at equivalent criteria* (characteristics), and procedure of receiving is *adoption of the optimum decision at equivalent criteria* (characteristics).

Under a condition, "***that***, … ***if***" the set of optimal solutions can be received. Receiving a set of optimal solutions and commitment (unique) solutions have a procedure for making optimal decisions at the equivalent criteria (characteristics).

5.CONCLUSIONS

The problem of optimal decision making in complex economic, technical systems on some set of functional characteristics is one of the most important problems of the system analysis and designing. In work are provided: a) new methods of vector optimization; b) methodology of creation of mathematical model of an economic system and its decision; c) methodology of creation of mathematical model of technical system in the conditions of definiteness and uncertainty in the form of a vector task of mathematical programming. In case of creation of characteristics in the conditions of uncertainty regression methods of transformation of information are used. The methodology of modeling and adoption of the optimal solution is based on normalization of criteria and the principle a maximine. This methodology has system nature and can be used when modeling both economic, and technical systems. Authors are ready to participate in the solution of vector problems of linear and nonlinear programming

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