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DISTANCE k-DOMINATION PARAMETER OF SOME GRAPHS AND ITS REALISATION

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ABSTRACT: ***A set D ⊆ V is called a distance k-dominating set of G if each vertex v ∈V(G) −D is within distance k from some vertex of D. The distance k-dominating set of the graph G is denoted by $D_k(G)$ and the distance k-domination number of G is denoted by $\gamma_k(G)$ is the minimum cardinality over all distance k-dominating sets. In this paper we established a general formulae for finding the k-dominating set of some graphs such as $(C_n)^r$, $(P_n)^r$, $(C_n \times P_m)$ and r^{th} power of the centipede graph with '2n' vertices.***

Keywords: Dominating set, Domination number, Distance k- dominating set, Tensor Product.

1.INTRODUCTION

Application of domination in graph lies in various fields in solving real life problems such as social network theory, land surveying, radio stations, computer communication networks, school bus routing, sets of representatives, interconnection networks etc. A graph is an ordered pair (V,E) where V is a non empty set of vertices and E is the set of edges, formed by pair of vertices. A subset D ⊆ V(G) is called a dominating set of G if every vertex in V(G) – D is adjacent to at least one vertex of D. The domination number γ(G) is the cardinality of the minimum dominating set of G.

Definition:1.1

A walk of a graph G is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n$ beginning and ending with vertices such that each edges e_i is incident with v_{i-1} and v_i. A walk is called a path if all its vertices are distinct. A path of n vertices is denoted by P_n.

Definition:1.2

A closed walk $v_0, v_1, v_2, \ldots, v_n = v_0$ in which n ≥ 3 and $v_0, v_1, v_2, \ldots, v_{n-1}$ are distinct is called a cycle. The graph consisting of a cycle having n vertices is denoted by C_n.

Definition:1.3

Let G and H be two graphs the tensor product of G and H is denoted by G × H the vertices of G × H is the cartesian product V(G)×V(H) and any two vertices (u,u') and (v,v') are adjacent in G × H iff u' is adjacent with v' and u is adjacent with v, for all u,v ∈G and u',v' ∈ H.

Definition:1.4

A subset D ⊆ V(G) is called a distance k-dominating set of G if every vertex in V(G) – D is within distance k from some vertex of D. The minimum cardinality among all distance k-dominating set of G is called the distance k-domination number denoted by γ_k(G).

Definition:1.5

Let G be any graph the r^{th} power of G is denoted by G^r and is obtained from G by joining all the vertices v_i ∈ G whose distance is at most r from each vertices of G.

Definition: 1.6

The n centipede is the tree on $2n$ nodes obtained by joining the bottoms of n copies of the path graph p_2 laid in a row with edges denoted by C_n .

Definition: 1.7

The floor and ceiling functions gives us the nearest integer up or down. The symbols for floor and ceiling are like the square brackets [ ] with the top or bottom part missing. $\lfloor x \rfloor$ $\lceil x \rceil$
floor (x) ceil (x).

Theorem: 1.8 Let P_n be a path graph with 'n' vertices. If G = $(P_n)^r$ then $\gamma_k(G) = \left\lceil \frac{n}{2rk+1} \right\rceil$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$.

Proof: Let P_n be a path graph with 'n' vertices. Let G = $(P_n)^r$.

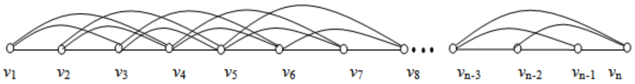

Figure 1.1 $(P_n)^3$

The vertex set of G is denoted by V = $\{v_1, v_2, v_3, \ldots, v_n\}$. Now divide the vertices of G in to m sets S_i, $1 \le i \le m$ such that each set consists of 2rk+1 vertices. Therefore

$S_1 = \{ v_1, v_2, \ldots, v_{2rk+1} \}$
$S_2 = \{ v_{2rk+2}, v_{2rk+3}, \ldots, v_{4rk+2} \}$
$S_3 = \{ v_{4rk+3}, v_{4rk+4}, \ldots, v_{8rk+3} \}$
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$S_{m-1} = \{ v_{2(m-2)rk+(m-1)}, \ldots, v_{2(m-1)rk+(m-1)} \}$
$S_m = \{ v_{2(m-1)rk+m}, \ldots, v_n \}$.

Case (i) If n ≡ 0 (mod 2rk+1), where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$ and k is the k-distance domination of G. Then $|S_i|$ = 2rk+1,for all $1 \le i \le m$, then collect all the middle vertices from each S_i, $1 \le i \le m$. Therefore, $D_k(G) = \{ v_{(2rk+1)\,i\,-\,rk} / 1 \le i \le m \}$

is the required k- dominating set of G

$| D_k(G) | = m$

$= \frac{n}{2rk+1}$

$= \left\lceil \frac{n}{2rk+1} \right\rceil$ [since n is a multiple of 2rk+1].

Case (ii) If $n \not\equiv 0 \pmod{2rk+1}$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$ and k is the k-distance domination of G. Then the last partition S_m containing less than 2rk+1 vertices, choose $p = \frac{2(m-1)rk+m+n}{2}$

Now, $D_k(G) = \{\{v_{(2rk+1)\,i\,-\,rk} / 1 \le i \le m\text{-}1\} \cup \{v_p\}\}$ is the required k- dominating set of G and

$| D_k(G) | = m - 1 + 1$

$= m$

$= \left\lceil \frac{n}{2rk+1} \right\rceil$

$\Rightarrow \gamma_k(G) = \left\lceil \frac{n}{2rk+1} \right\rceil$.

Theorem: 1.9 Let C_n be a cycle graph with 'n' vertices. If $G = (C_n)^r$ then $\gamma_k(G) = \left\lceil \frac{n}{2rk+1} \right\rceil$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$.

Proof: Let C_n be a cycle graph with 'n' vertices, Let $G = (C_n)^r$.

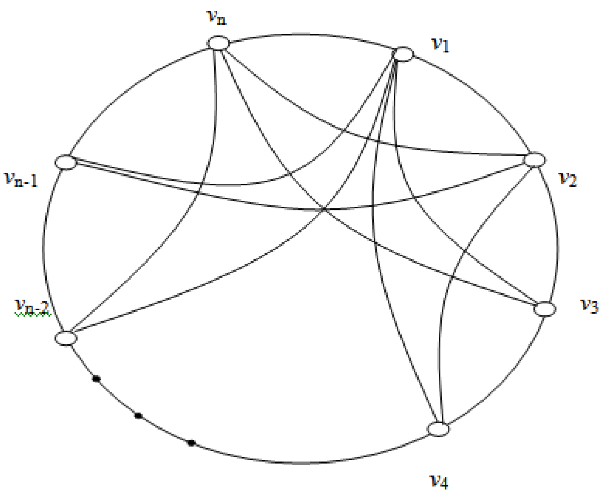

Figure 1.2 $(C_n)^3$

The vertex set of G is denoted by $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Each vertex $v_i \in G$ is adjacent with 2r vertices. Now divide the vertices of G in to m sets S_i, $1 \le i \le m$ such that each set consists of 2rk+1 vertices. Therefore,

$S_1 = \{ v_1, v_2, \ldots, v_{2rk+1} \}$
$S_2 = \{ v_{2rk+2}, v_{2rk+3}, \ldots, v_{4rk+2} \}$
$S_3 = \{ v_{4rk+3}, v_{4rk+4}, \ldots, v_{8rk+3} \}$
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$S_{m-1} = \{ v_{2(m-2)rk+(m-1)}, \ldots, v_{2(m-1)rk+(m-1)} \}$
$S_m = \{ v_{2(m-1)rk+m}, \ldots, v_n \}$.

Case (i) If $n \equiv 0 \pmod{2rk+1}$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$ and k is the k-distance domination of G. Then $|S_i| = 2rk+1$, for all $1 \le i \le m$, then collect all the middle vertices from each S_i, $1 \le i \le m$. Therefore, $D_k(G) = \{ v_{(2rk+1)\,i\,-rk} / 1 \le i \le m \}$ is the required k- dominating set of G and

$| D_k(G) | = m$

$= \frac{n}{2rk+1}$

$= \left\lceil \frac{n}{2rk+1} \right\rceil$ [since n is a multiple of 2rk+1].

Case (ii) If $n \not\equiv 0 \pmod{2rk+1}$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$ and k is the k-distance domination of G. Then the last partition S_m containing less than 2rk+1 vertices, choose $p = \frac{2(m-1)rk+m+n}{2}$

Now, $D_k(G) = \{\{v_{(2rk+1)i\,-\,rk} / 1 \le i \le m\text{-}1\} \cup \{v_p\}\}$ is the required k-dominating set of G and $| D_k(G) | = m - 1 + 1$

$= m$

$= \left\lceil \frac{n}{2rk+1} \right\rceil$

$\Rightarrow \gamma_k(G) = \left\lceil \frac{n}{2rk+1} \right\rceil$.

Result:1.10 Let $G = (C_n)^r$ be the graph with $\gamma_k(G) = \left\lceil \frac{n}{2rk+1} \right\rceil$ then $\gamma_k(G,p) = \gamma_k(G,q)$, where p=q=rk.

Proof: We know that $\gamma_k(G) = \left\lceil \frac{n}{2rk+1} \right\rceil$

$\gamma_k(G,p) = \left\lceil \frac{n}{2p+1} \right\rceil$ [since p = rk]

$= \left\lceil \frac{n}{2q+1} \right\rceil$ [since p = q]

$= \gamma_k(G,q)$.

Theorem: 1.11 Let $G = (C_n \times P_m)$, m > 1 be the graph with 'mn' vertices. Then the distance k-domination number of G is $\gamma_k(G) = \left\lceil \frac{n}{4k} \right\rceil$.

Proof: Let $G = (C_n \times P_m)$, m > 1 be the graph with 'mn' vertices is given in (figure 1.3).

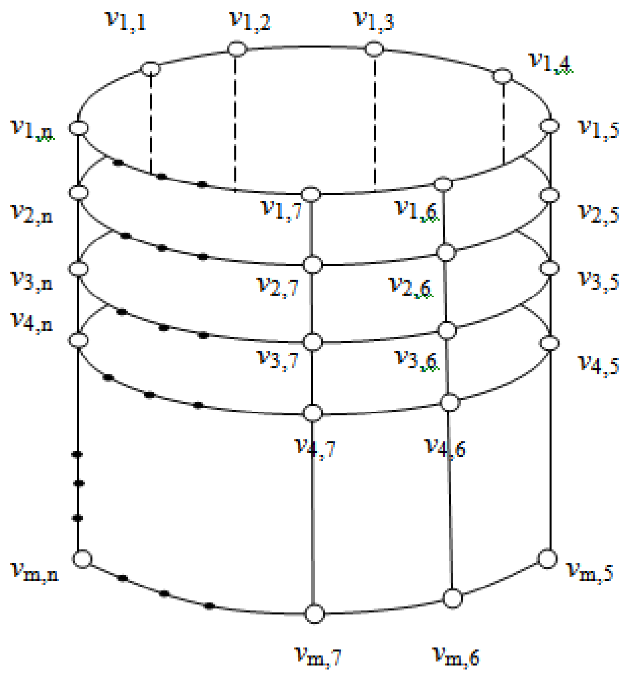

Figure 1.3

Now the vertex set of G is denoted by $V = \{v_{i,j} / 1 \le i \le m, 1 \le j \le n\}$. Each vertex $v_i \in G$ is adjacent with at most 4 vertices that is $d(v_{1,j}) = d(v_{m,j}) = 3, 1 \le j \le n$ and $d(v_{i,j}) = 4$, $2 \le i \le m\text{-}1$, $1 \le j \le n$. Now divide the vertices of G into m sets S_i, $1 \le i \le m$ such that

$S_1 = \{ v_{i,j} / 1 \le j \le n\}$
$S_2 = \{ v_{2,j} / 1 \le j \le n\}$
$S_3 = \{ v_{3,j} / 1 \le j \le n\}$
. . . .
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. . . .

$S_{m-1} = \{ v_{m-1,j} / 1 \le j \le n \}$
$S_m = \{ v_{m,j} / 1 \le j \le n \}$.

Case (i) If $n \equiv 0 \pmod{4k}$, where k is the k-distance domination of G. Then $|S_i| = n$, for all $1 \le i \le m$, then collect all the vertices from each S_i, $1 \le i \le m$ as follows: i) Let $t = \left\lceil \frac{n+1}{2} \right\rceil$. If m is odd then collect all vertices $\{ v_{i,t} / i = 1,3,5,\ldots,m\}$. ii) If m is even then collect all vertices $\{ v_{i,t+2k} / i = 2,4,6,\ldots,m\}$.
$\Rightarrow D_k(G) = \{\{ v_{m,k(4i-3)+1} / 1 \le i \le m$, for odd m$\} \cup \{ v_{m,k(4i-1)+1} / 1 \le i \le m$, for even m$\}\}$ is the required k-dominating set of G and $|D_k(G)| = \frac{m}{2} + \frac{m}{2}$

$= m$

$= \frac{n}{4k}$

$= \left\lceil \frac{n}{4k} \right\rceil$ [since n is a multiple of 4k].

Case (ii) If $n \not\equiv 0 \pmod{4k}$, where k is the k-distance domination of G. Then the last partition S_m containing less than n vertices, choose $p = \frac{1+n}{2}$

$\Rightarrow D_k(G) = \{\{ v_{m,k(4i-3)+1} / 1 \le i \le m$, for odd m$\} \cup \{ v_{m,k(4i-1)+1} / 1 \le i \le m$, for even m$\} \cup \{v_{mp}\}\}$ is the required k-dominating set of G and

$$|D_k(G)| = \frac{m-1}{2} + \frac{m-1}{2} + 1$$
$$= m$$
$$= \left\lceil \frac{n}{4k} \right\rceil$$

$\Rightarrow \gamma_k(G) = \left\lceil \frac{n}{4k} \right\rceil$.

Theorem:1.12 Let C_n be the centipede graph with '2n' vertices. If $G = (C_n)^r$ then $\gamma_k(G) = \left\lceil \frac{n}{2rk-1} \right\rceil$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$.

Proof: Let $G = (C_n)^r$ be the r^{th} power of the centipede graph with '2n' vertices.

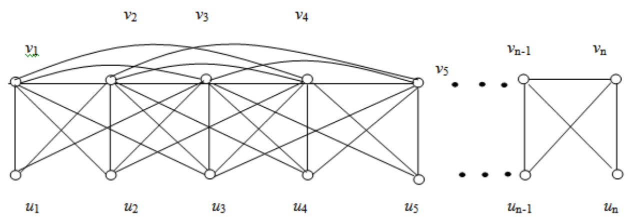

Figure 1.4 $(C_n)^3$

The vertex set of G is denoted by $V = V_1 \cup V_2$, where $V_1 = \{ v_i / 1 \le i \le n\}$, $V_2 = \{ u_i / 1 \le i \le n\}$. Now divide the vertices of V_1 into m sets S_i, $1 \le i \le m$ such that each set consists of 2rk-1 vertices. Therefore,

$S_1 = \{ v_1, v_2,\ldots,v_{2rk-1}\}$
$S_2 = \{ v_{(2rk-1)+1}, v_{(2rk-1)+2},\ldots,v_{4rk-2}\}$
$S_3 = \{ v_{(4rk-2)+1}, v_{(4rk-2)+2},\ldots,v_{6rk-3}\}$
. . . . .
. . . . .
. . . . .
$S_{m-1} = \{ v_{(m-2)(2rk-1)+1}, v_{(m-2)(2rk-1)+2},\ldots,v_{(m-1)(2rk-1)}\}$
$S_m = \{ v_{(m-1)(2rk-1)+1}, v_{(m-1)(2rk-1)+1},\ldots,v_n\}$.

Case (i) If $n \equiv 0 \pmod{2rk-1}$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$ and k is the k-distance domination of G. Then $|S_i| = 2rk-1$, for all $1 \le i \le m$, then collect all the middle vertices from each S_i, $1 \le i \le m$ Therefore, $D_k(G) = \{ v_{(2rk-1)i-(rk-1)} / 1 \le i \le m\}$ is the required k-dominating set of G and

$$|D_k(G)| = m$$
$$= \frac{n}{2rk-1}$$
$$= \left\lceil \frac{n}{2rk-1} \right\rceil \text{ [since n is a multiple of 2rk-1].}$$

Case (ii) If $n \not\equiv 0 \pmod{2rk-1}$, where $1 \le r \le \left\lceil \frac{n}{2} \right\rceil$ and k is the k-distance domination of G. Then the last partition S_m containing less than 2rk-1 vertices, choose $p = \frac{(m-1)(2rk-1)+1+n}{2}$

Now, $D_k(G) = \{\{v_{(2rk-1)i-(rk-1)} / 1 \le i \le m-1\} \cup \{v_p\}\}$ is the required k-dominating set of G and

$$|D_k(G)| = m - 1 + 1$$
$$= m$$
$$= \left\lceil \frac{n}{2rk-1} \right\rceil$$

$\Rightarrow \gamma_k(G) = \left\lceil \frac{n}{2rk-1} \right\rceil$.

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