

DETERMINATION OF VARIABLE SAMPLING PLANS FOR NON-NORMAL PROCESS THROUGH SKEWNESS AND KURTOSIS

D. RAMYA'S. DEVAARUL

1. Asst. Professor, Department of Statistics,
P.S.G. College of Arts and Science, Coimbatore.

2. Asst. Professor, Department of Statistics,
Government Arts College, Coimbatore.

ABSTRACT

In this paper contribution has been made in developing variable sampling plans for non-normal production process having log-logistic failure distribution. In industries many lifetime data are heavy tailed and follows non-normal pattern. The heavy tailed, log-logistic distribution has been used for analysis of sampling plans. In this paper, variable sampling plans and reliability sampling plans are developed and the efficiency measures such as operating characteristic OC function, ASN and AOQ have been provided. The variable sampling plans based on log-logistic distribution are designed through AQL and LQL with minimum angle technique. Tables are constructed for easy selection of the variable sampling plans to facilitate quality and reliability professionals.

Key words: Variable sampling plans, reliability sampling plans, log-logistic distribution, non-normal process, minimum angle method.

1. INTRODUCTION

Variable Sampling Plans play a vital role in product control measures through inspection of incoming lots. In the sampling plan literature, the measurable quality characteristic is assumed to be normally distributed. But in few circumstances the assumption is being violated due to target deviation of the process. To off-set the disadvantages, variable sampling plans are developed for a non-normal process. Log-logistic distribution is one of the most commonly used distributions for analysing skewed data. It is highly right skewed distribution. The cumulative distribution function of the log-logistic distribution could be written in a closed form and hence this distribution is particularly useful for analysis of survival data with censoring techniques. Therefore log-logistic distribution plays a vital role in analysing quality of the products and

hence can be used for developing sampling plans. In this article, the advantage of log-

logistic distribution is utilized in developing the sampling plans. The main advantage of log-logistic distribution is that the skewness and kurtosis depends on both the scale and shape parameters. Sommers [14] developed tables for the selection of variables double sampling plans and compared them with variables single sampling plan at two fixed points on the OC curve. Owen [10] developed variables sampling plans based on normal distribution for unknown standard deviation. However, the procedure for variable sampling plans based on non-normal processes has been suggested by authors like, Zimmer and Burr [16], Owen [11] and Takagi [15]. Kantam, SrinivasaRao and Sriram [7] proposed an economic reliability test plan for log-logistic distribution. Rosaiah, Kantam and Santosh Kumar [13] proposed a reliability test plan for exponentiated log-logistic distribution. Devaarul [4] developed mixed samplings plans and minimum tangent angle sampling plans. Devaarul and Jemmy Joyce [5] have developed reliability sampling plans based on minimum angle technique. Jemmy Joyce, Devaarul and Rebecca Edna [6] proposed mixed sampling plans based on the tangent angle, AQL and LQL. Kantam, Rosaiah, and Rao [8] studied the acceptance sampling based on life tests when the failure density model of the products is a log-logistic distribution. Ashkar [1] presented the fitting the Log-logistic Distribution by Generalized Moments. Bennet [3] proposed log-logistic regression models for survival data. Ragab and Green [12] proposed the properties of log-logistic distribution and also worked on the order statistics of the distribution.

1.1 FORMULATION OF THE SAMPLING PLANS

Let the random variable, 'X' is said to follow Log-logistic distribution.

The probability density function (pdf) is defined as:

$$f(x; \alpha, \beta) = \frac{\beta/\alpha(x/\alpha)^{\beta-1}}{(1+(x/\alpha)^\beta)^2} \quad (1)$$

The cumulative distribution function (cdf) of log-logistic distribution is

$$F(x; \alpha, \beta) = \frac{x^\beta}{x^\beta + \alpha^\beta}, \quad x > 0, \alpha > 0 \text{ and } \beta > 0 \quad (2)$$

where $\alpha > 0$ is the scale parameter and $\beta > 0$ is a shape parameter.

The log-logistic distribution is a continuous distribution and the shape of the log-logistic distribution is similar to that of log-normal distribution but it is heavy tailed distribution. This distribution is utilized in acceptance sampling plans and it is commonly used in survival analysis, hydrology and in networking.

1.2. CHARACTERISTICS OF THE LOG-LOGISTIC DISTRIBUTION

Bain [2] developed the Log-Logistic distribution and Ragab and Green [12] gave the properties of log-logistic distribution and also worked on the order statistics of log-logistic distribution. The various measures of the heavy tailed Log-logistic distribution are given below for easy references.

1.3. Mean: The mean of log-logistic distribution is denoted by \bar{x}

$$\bar{x} = \frac{\alpha \pi / \beta}{\text{Sin}(\pi / \beta)} \text{ for } \beta > 1, 0 \text{ otherwise.} \quad (3)$$

where $\alpha > 0$ is the scale parameter and $\beta > 0$ is a shape parameter and $\pi = 3.14$.

1.4. Variance: The variance of log-logistic distribution is denoted by S^2

$$S^2 = \alpha^2 \left(\frac{2 \pi / \beta}{\text{Sin} 2 \pi / \beta} - \frac{(\pi / \beta)^2}{\text{Sin}^2 \pi / \beta} \right) \quad (4)$$

1.5. Skewness: The skewness of log-logistic distribution is denoted by β_1

$$\beta_1 = \frac{\mu_3}{\mu_2^{3/2}} \quad (5)$$

Where μ_2 is the second order central moment and μ_3 is the third order central moment

1.6. Kurtosis: The kurtosis of log-logistic distribution is denoted by β_2

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \quad (6)$$

Where μ_2 is the second order central moment and μ_4 is the fourth order central moment

2. OPERATING PROCEDURE OF THE VARIABLE SAMPLING PLANS USING LOG-LOGISTIC VARIATE

Step 1: Draw a random sample of n items say $x_1, x_2, x_3, \dots, x_n$ from a lot of size N.

Step 2: Determine the Skewness (β_1) and Kurtosis (β_2) from the observed data and hence find the values of the scale parameter α and shape parameter β by using equations (5 and 6).

Step 3: Now obtain the corresponding mean \bar{x} and standard deviation S by using equations (3 and 4).

Step 4: If $\bar{x} + k\sigma \leq U$, accept the lot, otherwise reject it. (where \bar{x} is the sample mean and the standard deviation σ is known).

Step 5: In case of unknown standard deviation, if $\bar{x} + kS \leq U$, accept the lot, otherwise reject it.

2.1 DESIGNING PROCEDURE OF THE VARIABLE SAMPLING PLANS USING THE EXPANSION FACTOR (WHEN 'U' IS SPECIFIED)

In this section variable sampling plans indexed through AQL and LQL are determined. The procedure is as follows:

The statistics $Y = \bar{X} \pm kS$ are asymptotically normally distributed with the mean

$$\mu_Y = \mu \pm k\sigma \quad (7)$$

and the variance

$$\sigma_Y^2 = \frac{\sigma^2}{n} \left[1 + (k^2 / 4)(\beta_2 - 1) \pm k\beta_1 \right] \quad (8)$$

where β_1 and β_2 represent the skewness and kurtosis of the underlying distribution which is available in Takagi [15].

The underlying distribution is a Lomax distribution where λ is the scale parameter and α is the shape parameter.

Step 1: The OC function for unknown-sigma plan for a situation when U is specified is

$$P_a(p) = P[\bar{x} + k_U S \leq U | p] \quad (9)$$

Step 2:

$$K_{L_U(p)} = \frac{\sqrt{n_U} (k_U - K_p^*)}{\sqrt{e_U}} \quad (10)$$

where K_p^* is defined as

$$K_p^* = \frac{U - \mu}{\sigma} \quad (11)$$

Step 3: If U is specified determine the corresponding expansion factor e_U using the formula:

$$e_U = 1 + \frac{k_U^2}{4}(\beta_2 - 1) + k_U \beta_1 \quad (12)$$

Step 4: Then n_U and k_U values for unknown sigma plans are

$$n_U = e_U \left[\frac{K_\alpha + K_\beta}{K_{p_1}^* - K_{p_2}^*} \right]^2 \quad (13)$$

$$k_U = \frac{K_\alpha K_{p_2}^* + K_\beta K_{p_1}^*}{K_\alpha + K_\beta} \quad (14)$$

where K_α and K_β are the standardised deviates exceeded with the probabilities α and β respectively.

Step 5: The n' and k' values for known sigma plans are

$$n'_U = \frac{n_U}{e_U} \text{ and } k'_U = k_U \quad (15)$$

2.2. ALGORITHM FOR RELIABILITY SAMPLING PLANS USING WEIGHTED EXPANSION FACTOR

Step 1: Let 'L' be the lower specification, during the type I censoring with specified t_0 .

Step 2: Observe the sample of n items say $x_1, x_2, x_3, \dots, x_n$ from the testing field.

Step 3: Now determine the Skewness (β_1) and Kurtosis (β_2) from the observed data and hence find α using (5).

Step 4: Now obtain the corresponding mean \bar{x} and standard deviation σ using (3) and (4).

Step 5: If $\bar{x} - k\sigma \geq L$, accept the lot, otherwise reject it. (when the standard deviation σ is known).

Step 6: In case of unknown σ , if $\bar{x} - kS \geq L$, accept the lot, otherwise reject it.

3. DESIGNING PROCEDURE OF THE RELIABILITY SAMPLING PLANS USING THE EXPANSION FACTOR

Step 1: The OC function for unknown-sigma reliability sampling plan is defined as

$$P_a(p) = \Pr[\bar{X} - k_L S \geq L | p] \quad (16)$$

Step 2:

Also

$$K_{L_L(p)} = \frac{\sqrt{n_L}(k_L + K_{1-p}^*)}{\sqrt{e_L}} \quad (17)$$

where K_{1-p}^* is defined as

$$K_{1-p}^* = \frac{L - \mu}{\sigma}, 1 - p = \Pr(X > L) = \Pr(Z^* > K_{1-p}^*) \quad (18)$$

Step 3: For the known L, determine the corresponding expansion factor e_L using the formula:

$$e_L = 1 + \frac{k_L^2}{4}(\beta_2 - 1) - k_L \beta_1 \quad (19)$$

Step 4: The n_L and k_L values for unknown sigma plans are

$$n_L = e_L \left[\frac{K_\alpha + K_\beta}{K_{1-p_1}^* - K_{1-p_2}^*} \right]^2 \quad (20)$$

$$-k_L = \frac{K_\alpha K_{1-p_2}^* + K_\beta K_{1-p_1}^*}{K_\alpha + K_\beta} \quad (21)$$

where K_α and K_β are the standardised deviates exceeded with the probabilities α and β respectively.

Step 5: The n' and k' values for known sigma plans are

$$n'_L = \frac{n_L}{e_L} \text{ and } k'_L = k_L \quad (22)$$

3.1 EFFICIENCY MEASURES OF THE VARIABLE SAMPLING PLANS USING LOG-LOGISTIC DISTRIBUTION

3.1.1 Operating Characteristic function

The probability of accepting the lot of a sampling plan is given by the Operating Characteristic (OC) function.

Let 'X' follows log-logistic distribution with parameters (α, β)

The Lot will be accepted if the sample mean $\bar{x} \leq U - k\sigma$

The probability of acceptance of the lot is defined as

$$P_a(p) = P[\bar{x} + k\sigma \leq U] \\ = P[\bar{x} \leq U - k\sigma]$$

$$P_a(p) = \int_0^{U - k\sigma} f(\bar{x}) dx \quad (23)$$

Here

$$\bar{X} \rightarrow \text{Log - Logistic} \left(\frac{\alpha\pi/\beta}{\text{Sin}\pi/\beta}, \frac{S^2}{n} \right) \quad (24)$$

The proof for mean and variance of the sample mean in case of log-logistic distribution is given below

$$E(\bar{x}) = E\left(\frac{\sum x}{n}\right) = \frac{1}{n} E(\sum x) = \frac{1}{n} \sum E(x) = \frac{\sum \left(\frac{\alpha\pi/\beta}{\text{Sin}\pi/\beta}\right)}{n} = \frac{\alpha\pi/\beta}{\text{Sin}\pi/\beta} \quad (25)$$

$$V(\bar{x}) = V\left(\frac{\sum x}{n}\right) = \frac{1}{n^2} V(\sum x) = \frac{1}{n^2} \sum V(x) \quad (26)$$

$$= \frac{1}{n^2} \sum \left(\alpha^2 \left(\frac{2\pi/\beta}{\text{Sin} 2\pi/\beta} - \frac{(\pi/\beta)^2}{\text{Sin}^2 \pi/\beta} \right) \right) \\ = \frac{1}{n} \left(\alpha^2 \left(\frac{2\pi/\beta}{\text{Sin} 2\pi/\beta} - \frac{(\pi/\beta)^2}{\text{Sin}^2 \pi/\beta} \right) \right) = \frac{S^2}{n}$$

Hence Standard error is,

$$\sqrt{V(\bar{x})} = \frac{S}{\sqrt{n}} \quad (27)$$

3.2 Average Sample Number (ASN)

The Average Sample Number of this sampling plan is n.(i.e.,) $E(n) = n$.

Where

$$n = \left(\frac{S [K_{\alpha} + K_{\beta}]}{X_{p_1} - X_{p_2}} \right)^2 = \left(\frac{K_{\alpha} + K_{\beta}}{K_{p_1}^* - K_{p_2}^*} \right)^2 \quad (28)$$

(according to Takagi [15])

3.3. Average Outgoing Quality (AOQ)

The Average Outgoing Quality is given by

$$AOQ = p \cdot P_a(p) \left(\frac{N-n}{N} \right) \quad (29)$$

4. DESIGNING PROCEDURE BY MINIMUM ANGLE METHOD

Another designing procedure is that when the angle is minimized, then the abstract OC curve tends to ideal OC curve which leads to best sampling plans. In minimum angle technique generally a portion of the abstract OC curve is compared with the ideal OC curve. The approach of minimum angle method by considering the tangent of the angle between the lines joining the points (AQL, $1-\alpha$) and (LQL, β) is shown in Figure.1

$$\tan \theta = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)} \quad (30)$$

Thus when the two points on the OC curve are known, the minimum values of $\tan \theta$ can be calculated. This minimum angle provides a better sampling plan with good discriminating power. It minimizes the angle between the abstract and ideal OC curves. For a minimum $\tan \theta$, the angle θ approaches to zero and the chord AB approaches to AC, hence the ideal condition is reached. This approach minimizes both producers and consumers risk simultaneously. Thus both are benefitted by choosing these plans. The minimum angle method of variable sampling plans using Lomax Distribution is presented in Table 3. The parameters of variable sampling plans using Lomax distribution are chosen from Table 3 corresponding to the minimum angle.

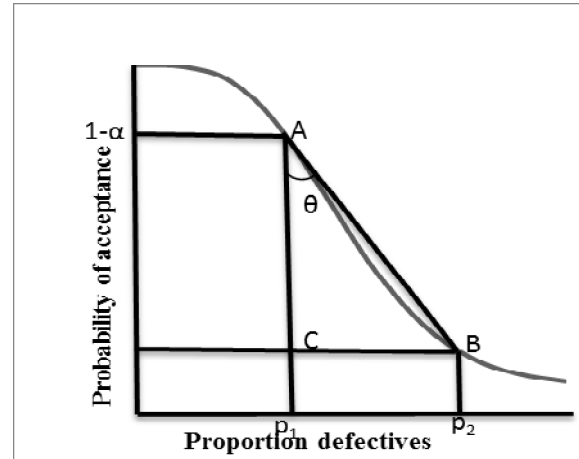


Figure 1: OC curve based on Minimum Tangent Angle

5. CONSTRUCTION OF TABLES

Table 1:

- i) The values of p_1 and p_2 are assumed to be known.
- ii) When U is specified the corresponding values of $K_{p_1}^*$ and $K_{p_2}^*$ are obtained from equation (11) (for $p = p_1$ and $p = p_2$).
- iii) The values of n_U and k_U for unknown sigma plans are obtained using equations (13 and 14).
- iv) Similarly the n'_U and k'_U values of known sigma plans are obtained using equation (15).

Table 2:

- i) The values of p_1 and p_2 are assumed to be known.
- ii) When L is specified the corresponding values of $K_{1-p_1}^*$ and $K_{1-p_2}^*$ are obtained from equation (18) (for $p = p_1$ and $p = p_2$).
- iii) The values of n_L and k_L for unknown sigma plans are obtained using equations (20 and 21). Similarly the n'_L and k'_L values of known sigma plans are obtained using equation (22).

Table 3:

- i) When U is specified the values of n_U , k_U , n'_U and k'_U are obtained using the equations (13,14 and 15) for known values of p_1 and p_2 .
- ii) The probability of acceptance is obtained using equation (9) for the values of p_1 and p_2 .
- iii) The corresponding minimum tangent angle is calculated using equation (30).

Table 4:

- i) When L is specified the values of n_L, k_L, n'_L and k'_L are obtained using the equations (20, 21 and 22) for known values of p_1 and p_2 .
- ii) The probability of acceptance is obtained using equation (16) for the values of p_1 and p_2 .
- iii) The corresponding minimum tangent angle is calculated using equation (30).

p_1	p_2	$K_{p_1}^*$	$K_{p_2}^*$	β_1	β_2	e_U	n_U	k_U	n'_U	k'_U
0.001	0.01	3.9299	2.5805	0.0871	1.2187	1.8262	9	3.1715	5	3.1715
0.002	0.02	3.5482	2.1859	0.1090	1.2293	1.7470	8	2.7825	5	2.7825
0.003	0.03	3.3285	1.9523	0.1308	1.2422	1.7294	8	2.5549	5	2.5549
0.004	0.04	3.1741	1.7838	0.1527	1.2575	1.7338	8	2.3926	4	2.3926
0.005	0.05	3.0559	1.6515	0.1746	1.2753	1.7493	8	2.2665	4	2.2665
0.006	0.06	2.9601	1.5420	0.1966	1.2955	1.7708	8	2.1630	4	2.1630
0.007	0.07	2.8790	1.4474	0.2186	1.3182	1.7957	8	2.0743	4	2.0743
0.008	0.08	2.8090	1.3640	0.2407	1.3434	1.8229	7	1.9968	4	1.9968
0.009	0.09	2.7475	1.2893	0.2628	1.3711	1.8515	7	1.9279	4	1.9279
0.01	0.1	2.6923	1.2213	0.2850	1.4014	1.8810	7	1.8655	4	1.8655
0.011	0.11	2.6427	1.1590	0.3073	1.4344	1.9112	7	1.8088	4	1.8088
0.012	0.12	2.5973	1.1011	0.3297	1.4701	1.9416	7	1.7563	4	1.7563
0.013	0.13	2.5550	1.0464	0.3522	1.5085	1.9716	7	1.7071	4	1.7071
0.014	0.14	2.5163	0.9955	0.3747	1.5497	2.0020	7	1.6615	4	1.6615
0.015	0.15	2.4796	0.9470	0.3974	1.5938	2.0317	7	1.6182	4	1.6182

Table 1: Values of n_U, k_U, n'_U and k'_U for Variable Sampling Plans based on log-logistic distribution using weighted expansion factor (When 'U' is specified)

p_1	p_2	$K_{1-p_1}^*$	$K_{1-p_2}^*$	β_1	β_2	e_L	n_L	k_L	n'_L	k'_L
0.001	0.01	-3.9299	-2.5805	0.0871	1.2187	1.2736	6	3.1715	5	3.1715
0.002	0.02	-3.5482	-2.1859	0.1090	1.2293	1.1407	5	2.7825	5	2.7825
0.003	0.03	-3.3285	-1.9523	0.1308	1.2422	1.0611	5	2.5549	5	2.5549
0.004	0.04	-3.1741	-1.7838	0.1527	1.2575	1.0032	4	2.3926	4	2.3926
0.005	0.05	-3.0559	-1.6515	0.1746	1.2753	0.9578	4	2.2665	4	2.2665
0.006	0.06	-2.9601	-1.5420	0.1966	1.2955	0.9204	4	2.1630	4	2.1630
0.007	0.07	-2.8790	-1.4474	0.2186	1.3182	0.8888	4	2.0743	4	2.0743
0.008	0.08	-2.8090	-1.3640	0.2407	1.3434	0.8617	4	1.9968	4	1.9968
0.009	0.09	-2.7475	-1.2893	0.2628	1.3711	0.8381	3	1.9279	4	1.9279
0.01	0.1	-2.6923	-1.2213	0.2850	1.4014	0.8175	3	1.8655	4	1.8655
0.011	0.11	-2.6427	-1.1590	0.3073	1.4344	0.7994	3	1.8088	4	1.8088
0.012	0.12	-2.5973	-1.1011	0.3297	1.4701	0.7835	3	1.7563	4	1.7563
0.013	0.13	-2.5550	-1.0464	0.3522	1.5085	0.7693	3	1.7071	4	1.7071
0.014	0.14	-2.5163	-0.9955	0.3747	1.5497	0.7568	3	1.6615	4	1.6615
0.015	0.15	-2.4796	-0.9470	0.3974	1.5938	0.7457	3	1.6182	4	1.6182

Table 2: Values of n_L, k_L, n'_L and k'_L for the Reliability sampling plans based on log-logistic distribution using weighted expansion factor

p_1	p_2	β_1	β_2	n_U	k_U	n'_U	k'_U	$P_a(p_1)$	$P_a(p_2)$	$\tan \theta$
0.001	0.01	0.0871	1.2187	9	3.1715	5	3.1715	1.0000	0.9951	1.8404
0.002	0.02	0.1090	1.2293	8	2.7825	5	2.7825	0.9998	0.9856	1.2662
0.003	0.03	0.1308	1.2422	8	2.5549	5	2.5549	0.9996	0.9745	1.0793
0.004	0.04	0.1527	1.2575	8	2.3926	4	2.3926	0.9992	0.9628	0.9868
0.005	0.05	0.1746	1.2753	8	2.2665	4	2.2665	0.9989	0.9507	0.9338
0.006	0.06	0.1966	1.2955	8	2.1630	4	2.1630	0.9985	0.9385	0.9000
0.007	0.07	0.2186	1.3182	8	2.0743	4	2.0743	0.9980	0.9261	0.8762
0.008	0.08	0.2407	1.3434	7	1.9968	4	1.9968	0.9975	0.9137	0.8592
0.009	0.09	0.2628	1.3711	7	1.9279	4	1.9279	0.9970	0.9014	0.8469
0.01	0.1	0.2850	1.4014	7	1.8655	4	1.8655	0.9965	0.8890	0.8377
0.011	0.11	0.3073	1.4344	7	1.8088	4	1.8088	0.9959	0.8768	0.8311
0.012	0.12	0.3297	1.4701	7	1.7563	4	1.7563	0.9953	0.8646	0.8261
0.013	0.13	0.3522	1.5085	7	1.7071	4	1.7071	0.9947	0.8523	0.8218
0.014	0.14	0.3747	1.5497	7	1.6615	4	1.6615	0.9941	0.8403	0.8192
0.015	0.15	0.3974	1.5938	7	1.6182	4	1.6182	0.9934	0.8282	0.8170

Table 3: The values of the sample size n, acceptance constant k, tanθ for the known and unknown sigma plans (when 'U' is specified), for given values of p_1 and p_2 – Variable Sampling Plans based on log-logistic distribution

p_1	p_2	β_1	β_2	n_L	k_L	n'_L	k'_L	$P_a(p_1)$	$P_a(p_2)$	$\tan \theta$
0.006	0.06	0.1966	1.2955	4	2.1630	4	2.1630	1.0000	0.9945	9.8652
0.007	0.07	0.2186	1.3182	4	2.0743	4	2.0743	0.9999	0.9928	8.8200
0.008	0.08	0.2407	1.3434	4	1.9968	4	1.9968	0.9999	0.9910	8.0274
0.009	0.09	0.2628	1.3711	3	1.9279	4	1.9279	0.9999	0.9890	7.4031
0.01	0.1	0.2850	1.4014	3	1.8655	4	1.8655	0.9999	0.9868	6.8950
0.011	0.11	0.3073	1.4344	3	1.8088	4	1.8088	0.9999	0.9846	6.4746
0.012	0.12	0.3297	1.4701	3	1.7563	4	1.7563	0.9998	0.9822	6.1172
0.013	0.13	0.3522	1.5085	3	1.7071	4	1.7071	0.9998	0.9796	5.8012
0.014	0.14	0.3747	1.5497	3	1.6615	4	1.6615	0.9998	0.9770	5.5329
0.015	0.15	0.3974	1.5938	3	1.6182	4	1.6182	0.9997	0.9742	5.2905

Table 4: The values of the sample size n, acceptance constant k, tanθ for the known and unknown sigma for Reliability Sampling Plans, given values of p_1 and p_2 – Reliability Sampling Plans based on log-logistic distribution.

5.1 SELECTION OF SAMPLING PLANS

For a production process when U is specified, and provided with ($p_1=0.001, 1-\alpha = 0.95$), ($p_2 = 0.01, \beta = 0.10$). Also the process is observed to follow a Lomax distribution with the corresponding $\beta_1 = 0.0871$ and $\beta_2 = 1.2187$. Determine the corresponding variable sampling plan parameters with minimum tangent angle.

Solution:

Since the production process has $\beta_1 = 0.0871$ and $\beta_2 = 1.2187$, from Table 4, it can be observed that when the process σ is known then $n = 9, k=3.1715$. When the process σ is unknown then $n' = 5, k' = 3.1715$ and the corresponding angle $\tan\theta = 1.8404$

6. CONCLUSION

In variable sampling plans, many processes are based on normal distribution. But in most of the practical situations the process is non-normal and heavy tailed. In a heavy tailed distribution, the sample statistic will have a large variance and the sample mean generally underestimates the population mean. Also the sample mean of a heavy tailed distribution does not follow a normal distribution, even after a large number of samples. Hence this paper provides an insight to sampling plans based on non-normal distributions. Since variable sampling plans are the base for designing reliability sampling plans, the procedure for developing non-normal reliability sampling plans are also discussed in this paper. The sampling plans for the non-normal distribution are developed through an expansion factor, which is a function of skewness and kurtosis. The skewness and kurtosis play a vital role in the heavy tailed distribution. The designing and operating procedure are provided by utilizing skewness and kurtosis. The tables are constructed for easy selection of the plans. A procedure for minimizing the angle between the AQL and LQL values are also provided in this thesis. This makes the consumer and producer to easily select the exact variable sampling plans. Similarly for any non-normal process this method could be employed and easily implemented in industries. The operating and designing procedures with necessary tables for easy selection of the sampling plans are provided. It is found that by applying the minimum angle technique, there is a great reduction in the sample size. This minimizes the risk of both the producer and consumer. The minimum tangent angle method helps to choose a better sampling plan which may tend towards an ideal situation. Also, it provides a better discrimination power of accepting good lots.

REFERENCES

- [1] Ashkar, F. and Mahdi, S., "Fitting the Log-logistic Distribution by Generalized Moments", *Journal of Hydrology*, Vol.328, pp.694-703, 2006.
- [2] Bain, L.J., "Analysis for the linear failure rate life-testing distribution", *Technometrics*, Vol.16 (4), pp.551 – 559, 1974.
- [3] Bennet, S., "Log-logistic Regression Models for Survival Data", *Journal of the Royal Statistical Society*, Series C, 32(2), 165-171, 1983.
- [4] Devaarul, S., "Certain Studies Relating to Mixed Sampling Plans and Reliability based Sampling Plans", Ph.D., Thesis, Department of Statistics, Bharathiar University, Coimbatore, Tamil Nadu, India, 2002.
- [5] Devaarul, S. and Jemmy Joyce, V., "Designing and Selection of Reliability Sampling Plans Based on Minimum Angle Technique", *International Journal of Mathematics and Computation*, Vol. 20, no.3, pp.60-65, 2013.
- [6] Jemmy Joyce, V., Devaarul, S. and Rebecca Jebaseeli Edna, K., "Designing and Selection of Mixed Sampling Plans based on Tangent Angle", *International Journal of Mathematics and Computer Applications Research*, Vol.3, Issue 1, pp.217-222, 2013.
- [7] Kantam, R.R.L. and SrinivasaRao, G. and Sriram, G., "An economic reliability test plan: log-logistic distribution", *Journal of Applied Statistics*, Vol. 33(3), pp.291-296, 2006.
- [8] Kantam, R.R.L., Rosaiah, K. and Rao, G.S, "Acceptance Sampling based on Life Tests: Log-Logistic Models", *Journal of Applied Statistics*, Vol.28, pp. 121-128, 2001.
- [9] Montgomery, D.C., "The Effect of Non-normality on Variables Sampling Plans", *Naval Research Logistics Quarterly*, Vol.32, pp.27-33, 1985.
- [10] Owen, D. B., "Variables Sampling Plans Based on the Normal Distribution", *Technometrics*, Vol. 9, pp.417–423, 1967.
- [11] Owen, D.B., "Factors for One-Sided Tolerance Limits and for Variables Sampling Plans", SCR-607, Sandia Corporation monograph, 1963.
- [12] Ragab, A. and Green, J., "On order statistics from the Log-logistic distribution and their properties", *Communications in Statistics - Theory and Methods*, Vol.13 (21), pp.2713- 2724, 1984.
- [13] Rosaiah, K., Kantam, R. R. L. and Santhosh Kumar, C., "Reliability Test Plans for Exponentiated Log-Logistic Distribution", *Economic Quality Control*, Vol. 21(2), pp.165-175, 2006.
- [14] Sommers, D. J., "Two-Point Double Variables Sampling Plans", *Journal of Quality Technology*, Vol. 13, pp.25-30, 1981.
- [15] Takagi, k., "On Designing Unknown-sigma Sampling Plans based on a Wide Class of Non-normal Distributions", *Technometrics*, Vol. 14, No.3, pp. 669-678, 1972.

Zimmer, W.J. and Burr, I.W., "Variables Sampling Plans Based on Non- normal Populations", *Industrial Quality Control*, Vol.20, No.1, pp.18–26, 1963.