

PERIODIC SHAPING THE OZONE LAYER AND THE ULTRAVIOLET RAYS

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Abstract: In this work, an analysis of the current situation between the ozone layer and ultraviolet rays is made, indicating the dangers that the decrease in the density of the ozone layer represents by the unconscious activity of the homes; the problems that global warming represents for humanity, as well as the thawing of the poles, are indicated. If this process is simulated using a system of differential equations periodic in general; using Floquet's theory the system is reduced to a system where the matrix of the linear part of the system has constant coefficients and the non-linear part is periodic with respect to time; examples are placed that satisfy the theoretical conditions indicated, and the trajectories of the corresponding system are plotted.

Keywords: Environment, ozone, ultraviolet rays.

I. INTRODUCTION

The greenhouse effect is a natural phenomenon caused by the concentration of gases in the atmosphere, which form a layer that allows the passage of sunlight and the absorption of heat. This process is responsible for keeping the Earth at an appropriate temperature, ensuring the necessary heat; without it, life on our planet would be impossible, as the cold would prevent survival [9], [10], [13].

After the United Nations Conference on the Environment, in June 1972, in Stockholm, the world focused on environmental preservation. Various sectors, companies, governments, environmentalists, organized civil society, among others, discussed in depth the subject that created a sense or unison of discomfort, that is, it has been the insomnia of many people across the planet and it is not for less, since the effects of imbalance emerge in every place imaginable, frightening and hopeless.

The situation of the environment is increasingly a more current problem [11]. UN Secretary-General Antonio Guterres urged statesmen not to take the podium without "concrete and transformative plans" to stop rising global temperatures, achieve carbon neutrality and reduce carbon emissions by 45%. It is not a summit to speak, we talk a lot; it is not a summit to negotiate, because it is not negotiated with nature. It is an action summit, governments have come to show how committed they are, who the leaders are to invest in a green future. Above all, young

people are demanding that they act urgently and be right," said the Secretary-General. 17-year-old Greta Thunberg was strict: "They stole my dreams, my hope, with their empty words. The only thing they talk about is money and they tell us stories about perpetual economic growth. How dare they? I want them to panic, to feel the fear that I feel every day and after they act, let them act as if the house is on fire".

In [11] it is indicated, "the moment in which we live, in a chaotic and alarming situation, driven by scientific and technological advances, which enabled and stimulated new classes of consumption and new markets. Most of which occurred without the mechanisms for the preservation or recovery of the environment having to be structured or contemplated, not only to improve the quality of life of the present generations, but mainly, in order to make possible the existence of future generations".

In the work [16] entitled Ecological Literacy: a discussion on the philosophical and sociological aspects of environmental education as considerations: "A human civilization and its consumer culture, driven in recent years by the advent of technology, led to a devastating process of its fundamental ecosystems and, consequently, a crisis in a society, such as economic, social, educational and why not philosophical".

Every pollution and depollution process is directly related to the information maintained by society, whatever it may be. [5] the concern to be able to contribute positively is expressed: "It was necessary for the avalanche of information of the most diverse types and by the most diverse means with which we are confronted to better understand that information is only a necessary condition of knowledge. Perhaps the more perverse is that the construction of knowledge is as easy as the current access to information through the simple press of a key".

In [12], the structural differences between developed and underdeveloped countries are pointed out in terms of the impact and differentials brought by the new way of exploring the environment combined with the new initiative that focuses on the policy of industrialization and development of economic and social structures, because inequalities in the appropriation of resources between

countries and between groups within countries are conflicting.

The ozone layer is inside the Earth's atmosphere and has the function of being a protective layer that accurately preserves the life of the planet Earth, acting as a shield against the sun's rays called ultraviolet radiation, absorbing 97 to 99% of it. It is located at a distance from the Earth's surface with 15 to 50 kilometers in height in greater concentration, although it is also present in the ground [14].

Carbon dioxide is essential to life on the planet, since it is one of the essential compounds for the realization of photosynthesis - the process by which photosynthetic organisms transform solar energy into chemical energy. This chemical energy, in turn, is distributed to all living beings through the food web. This process is one of the phases of the carbon cycle and is vital for the maintenance of living beings.

Carbon is a basic element in the composition of organisms, making it indispensable for life on the planet. This element is stored in the atmosphere, in the oceans, soils, sedimentary rocks and is present in fossil fuels. However, carbon is not fixed in any of these stocks. There are a series of interactions through which carbon is transferred from one stock to another. Many organisms in terrestrial and ocean ecosystems, such as plants, absorb the carbon found in the atmosphere in the form of carbon dioxide (CO₂). This absorption occurs through the process of photosynthesis. On the other hand, the various organisms, both plants and animals, release carbon dioxide into the atmosphere through the process of respiration. There is also the exchange of carbon dioxide between the oceans and the atmosphere through diffusion. Thanks to some products generated by humans and called halocarbons, the destruction of the ozone layer has accelerated compared to its natural rhythm.

This causes the thinning of the layer and the generation of the known ozone holes, with which the Earth loses protection against solar radiation. The passage of the strongest solar rays causes diseases in human life such as skin cancer or cataracts in the eyes. Faced with this problem, the UN (United Nations), on September 16, 1987, signed the Montreal Protocol and, in 1994, the United Nations General Assembly declared on September 16 the International Day for the Preservation of the Ozone Layer.

Severe wear and tear of the ozone layer will lead to an increase in cases of melanomas, skin cancer, eye cataracts, suppression of the immune system in humans and other species. Thanks to some products generated by humans and called halocarbons, the destruction of the ozone layer has accelerated compared to its natural pace. This causes the thinning of the layer and the generation of the known ozone holes, with which the Earth loses protection against solar radiation. The passage of the strongest sun rays causes diseases in human life such as skin cancer or cataracts in the eyes [15], [18].

The Russian Air Observatory has warned that the thickness of the ozone layer in the Arctic is close to a critical limit. In late February, the ozone layer in northern eastern Siberia was reduced by between 30 and 40 Dobson

units. The observatory attributes the weakening to the formation of a polar storm recorded this year in the stratosphere with temperatures below 78 degrees below zero. Scientists recalled that the weakening of the ozone layer in mid-2011 caused an increase in ultraviolet radiation in many areas of the northern hemisphere.

Among the many books dedicated to mathematical modeling, four [6], [7] and [8] will be indicated in which real problems are simulated by means of differential equations and systems of equations, where in addition a certain treatment is done to give conclusions of the processes. In [8] the authors simulate the shape of the process of polymer formation in the blood using autonomous systems of differential equations of third and fourth order, giving conclusions on the formation of polymers and domains.

In [7] different real-life problems are treated using equations and systems of differential equations, all of them only in the autonomous case; where examples are developed, and other problems and exercises are presented for them to be developed by the reader. The authors of [6] indicate a set of articles forming a collection of several problems that are modeled in different ways, but in general the qualitative and analytical theory of differential equations is used in both autonomous and non-autonomous cases.

In [1] and [2] the process of eliminating liquid pollution in oxidation ponds was simulated, using a system of differential equations with constant coefficients, but in general the pollution additions occur periodically, making a model in the case not autonomous and especially periodic in relation to time; in [3] and [17] the simulation is done by means of non-autonomous systems and in particular periodicals; this is a specific example of a compartmental model.

II. MODEL FORMULATION

Using what is really going on, it is evident that the model will not be able to converge to acceptable values of the density of the oxygen layer and the infrared rays. The variation in the density of the oxon layer is proportional to its density and decreases with the addition of infrared rays; likewise, with the increase in the density of the oxon layer, the infrared rays decrease and increase with the increase of its density.

For the formulation of the model we will denote by, \tilde{x}_1 is the total density of the oxygen layer at the moment t . \tilde{x}_2 is the total density of the infrared rays at the moment t . In addition, it will be denoted by

$$V_\alpha = \{(\tilde{x}_1, \tilde{x}_2) \in R^2 / a - \alpha < \tilde{x}_1 < a + \alpha, b - \alpha < \tilde{x}_2 < b + \alpha\}$$

The set of values for the permissible density of the oxygen layer and the infrared rays respectively.

Here the variables will be introduced x_1 and x_2 defined as follows: $x_1 = \tilde{x}_1 - \bar{x}_1$ and $x_2 = \tilde{x}_2 - \bar{x}_2$ so if $\tilde{x}_1 \rightarrow 0$ and $\tilde{x}_2 \rightarrow 0$ the following conditions would be met $\tilde{x}_1 \rightarrow \bar{x}_1$ and $\tilde{x}_2 \rightarrow \bar{x}_2$, which would constitute the main objective of this work. In this way the model will be given by the following system of equations,

$$\begin{cases} x_1' = a_1x_1 - a_2x_2 + X_1(x_1, x_2) \\ x_2' = -a_3x_1 + a_4x_2 + X_2(x_1, x_2) \end{cases} \quad (1)$$

Where $X_i(x_1, x_2)$, ($i = 1, 2$) they are disturbances not inherent in the process, which could at a given moment produce certain changes; and from a mathematical point of view they are infinitesimals of a higher order, those that admit the following development in series of potentials,

$$X_i(x_1, x_2) = \sum_{|p| \geq 2} X_i^p x_1^{p_1} x_2^{p_2} \quad (i = 1, 2), |p| = p_1 + p_2$$

It is evident that this is an unstable system and therefore the total density would not converge to admissible values, than the existing situation due to the politicians' indifference towards the environment; to achieve our objective, it is necessary to change the existing reality, that is, to make sure that the ozone layer does not decrease with the presence of infrared rays, and that the infrared rays do not grow in proportion to their concentration; here would be the action of the home for the protection of nature, Thus certain controls would appear that would regulate the process, falling as follows.

$$\begin{cases} x_1' = a_1x_1 + (b_1 - a_2)x_2 + X_1(x_1, x_2) \\ x_2' = -a_3x_1 + (a_4 - b_2)x_2 + X_2(x_1, x_2) \end{cases}$$

The aforementioned control is associated with the home's action to provide solutions to the environmental problems that currently appear; this way of acting is associated with the search for clean and renewable energy, among which the following can be listed: wind energy, solar panels, biogas, among other actions.

This system can be written in the form,

$$\begin{cases} x_1' = ax_1 + bx_2 + X_1(x_1, x_2) \\ x_2' = -cx_1 - dx_2 + X_2(x_1, x_2) \end{cases} \quad (2)$$

The characteristic equation of the matrix of the linear part of the system (1) has the following form,

$$\begin{vmatrix} a - \lambda & b \\ -c & -d - \lambda \end{vmatrix} = 0$$

This expression is equivalent to,

$$\lambda^2 + (d - a)\lambda + (bc - ad) = 0$$

In this case, applying the first approximation method, the following result is obtained.

Theorem1: The null solution of the system (2) is asymptotically stable if and only if the following conditions are met: $d > a$ and $bc > ad$.

The proof is a direct consequence of the conditions of Hurwitz's theorem.

Note1: If the conditions: $d > a$ and $bc > ad$, are satisfied, then the densities of the oxygen layer and the infrared rays converge to allowable values, otherwise it is necessary to add our action to prevent environmental catastrophes.

As in general, the processes of aggression towards nature appear periodically, for example the fires that produce deforestation are generally present in the quench months, in addition to other phenomena that have these characteristics; so it will be considered that all the coefficients depend on the periodic form of time, so the system will have the form,

$$\begin{cases} x_1' = a(t)x_1 + b(t)x_2 + X_1(t, x_1, x_2) \\ x_2' = -c(t)x_1 - d(t)x_2 + X_2(t, x_1, x_2) \end{cases} \quad (3)$$

Here Floquet's theory [4] will be applied to transform the system (3) into a system where the matrix of the linear part has constant coefficients, and where non-linear functions are ω - periodic with respect to time.

In this case the system (3) can be written in vector form, $x' = A(t) + X(t, x)$ (4)

Where $x = col(x_1, x_2)$, $A(t) = \begin{bmatrix} a(t) & -b(t) \\ c(t) & -d(t) \end{bmatrix}$ and

$X(t, x) = col[X_1(t, x_1, x_2), X_2(t, x_1, x_2)]$, such that $A(t + \omega) = A(t)$, $X(t + \omega, x) = X(t, x)$.

With the associated Linear system,

$$x' = A(t)x \quad (5)$$

Be $\Phi(t)$ the fundamental matrix of the system (5) and B a constant matrix related to the fundamental matrix as follows, $\Phi(t + \omega) = \Phi(t)B$ in such a way that there is R , such that $R = \omega^{-1} \ln B$, and so the function is defined $G(t) = \Phi(t)e^{-Rt}$; these functions will be used in the fundamental result of this work.

Theorem2: There is a coordinate transformation,

$$x = G(t)y \quad (6)$$

which reduces the system (4) in the next system,

$$y' = Ry + Y(t, y) \quad (7)$$

Demonstration: Deriving the transformation (6) along the trajectories of the systems (4) and (7) we have the following system of equations,

$$x' = \Phi'(t)e^{-Rt} - R\Phi(t)e^{-Rt}y + \Phi(t)e^{-Rt}y' \quad (8)$$

Replacing x' and Φ' in (8) and considering that x and Φ are solutions of (4) and (5) respectively and furthermore using (6) the following expression is reached:

$$\begin{aligned} A(t)\Phi(t)e^{-Rt}y + X[t, \Phi(t)e^{-Rt}y] \\ = A(t)\Phi(t)e^{-Rt}y - R\Phi(t)e^{-Rt}y \\ + \Phi(t)e^{-Rt}y' \end{aligned}$$

Reducing similar terms if you have,

$$X[t, \Phi(t)e^{-Rt}y] = -R\Phi(t)e^{-Rt}y + \Phi(t)e^{-Rt}y'$$

Now isolating y' in the previous expression you must,

$$y' = Ry + \Phi(t)^{-1}e^{Rt}X[t, \Phi(t)e^{-Rt}y]$$

Thus, it is concluded that,

$$y' = Ry + Y(t, y)$$

Where $Y(t, y) = \phi(t)^{-1} e^{Rt} X[t, \phi(t) e^{-Rt} y]$, thus showing the theorem.

Eigenvalues μ_1, μ_2 of the matrix B , system multipliers (5) are called, however the eigenvalues λ_1, λ_2 matrix R are the characteristic indexes of this system. From the definitions of matrices B and R , the following relationship is satisfied,

$$\lambda_1 = \omega^{-1} \ln \mu_1, \lambda_2 = \omega^{-1} \ln \mu_2.$$

By defining the Neperian logarithm of a complex number, one must,

$$\ln \mu_i = \ln |\mu_i| + i(\arg \mu_i + 2\pi), i = 1, 2.$$

Theorem3: If $0 < |\mu_i| < 1, i = 1, 2$, it is deduced that, $Re \lambda_i < 0, i = 1, 2$ and this implies that the equilibrium position of the system (5) and therefore that of the system (4) is asymptotically stable.

Demonstration: As a consequence of theorem2, the system (4) is equivalent to the system (7) and the proper values of the matrix R , that is to say, λ_1, λ_2 are such that, $\lambda_1 = \omega^{-1} \ln \mu_1, \lambda_2 = \omega^{-1} \ln \mu_2$, and why $\ln \mu_i = \ln |\mu_i| + i(\arg \mu_i + 2\pi), i = 1, 2$, if you have to, $Re \lambda_i = \omega^{-1} \ln |\mu_i| < 0$, because $0 < |\mu_i| < 1, i = 1, 2$. This completes the demonstration.

Example1: Suppose that in the dynamic process between the oxygen layer and the infrared rays, the variation of the oxygen layer does not depend significantly on the infrared rays; and that likewise the variation of the latter does not depend significantly on the concentration of the oxygen layer; so the model will have the shape,

$$\begin{cases} x_1' = (cost + n)x_1 + X_1(t, x_1, x_2) \\ x_2' = -(sent + p)x_2 + X_2(t, x_1, x_2) \end{cases} \quad (9)$$

Where n and p are real numbers and the functions X_1 and X_2 contain higher-grade terms with respect to x_1 and x_2 , and are 2π -periodic with respect to time.

In this case, the fundamental matrix has the following form:

$$\phi(t) = \begin{bmatrix} e^{nt+sent} & 0 \\ 0 & e^{-pt+cost} \end{bmatrix}$$

The constant matrix B such that:

$$\phi(t + 2\pi) = B\phi(t)$$

It has the shape,

$$B = \begin{bmatrix} e^{2\pi n} & 0 \\ 0 & e^{-2\pi p} \end{bmatrix}$$

In this way, if you have the eigenvalues of B , the multipliers of the system (9) are,

$$\mu_1 = e^{2\pi n} \quad \mu_2 = e^{-2\pi p}$$

And therefore, the eigenvalues of R , that is to say the characteristic indices of the system (9) have the form,

$$\lambda_1 = (2\pi)^{-1} \ln e^{2\pi n} = n, \lambda_2 = (2\pi)^{-1} \ln e^{-2\pi p} = -p.$$

It follows that the stability of the transformed system will depend on the signal of n and p ,

$$\begin{cases} x_1' = \lambda_1 x_1 + X_1(t, x_1, x_2) \\ x_2' = \lambda_2 x_2 + X_2(t, x_1, x_2) \end{cases}$$

This example illustrates the previous results concretely, since λ_1, λ_2 are such that your signal will depend on n and $-p$, but it will depend on the range in which the values of μ_1, μ_2 .

Below we present the example that will be developed in a computational way as an illustration of the theory studied and that comply with the conditions of the previous example.

Example2: Let the system with periodic coefficients follow, which simulates the process of dynamic insulin glucose for a healthy patient.

$$\begin{cases} x_1' = (cost - 1)x_1 - (cost + 3)x_1^3 x_2^2 \\ x_2' = -(sent + 2)x_2 - (sent + 1)x_1^2 x_2^3 \end{cases}$$

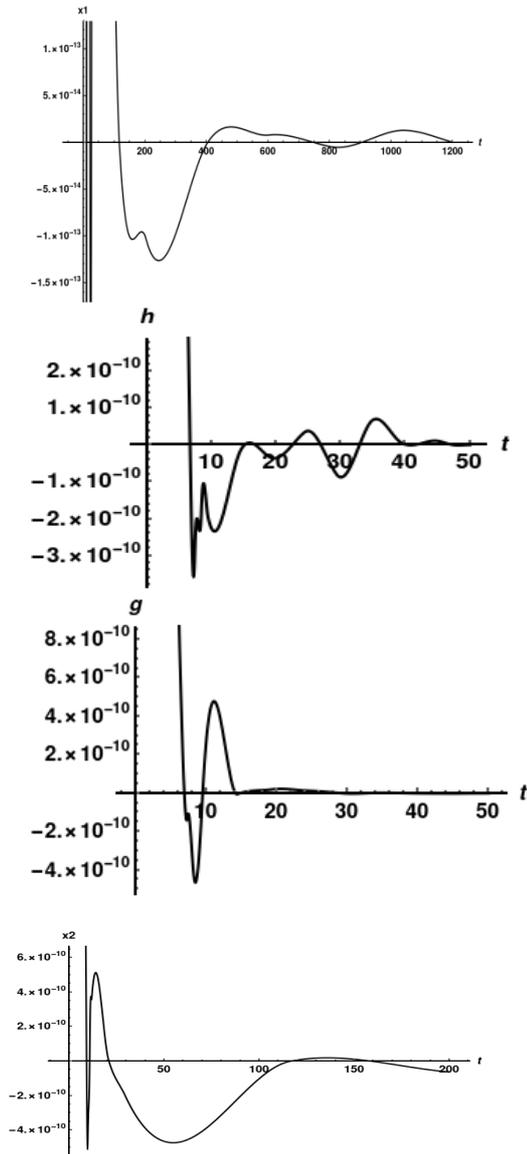


Fig. 1. Graph x_1 over time. Fig. 2. Graph of x_2 in time. The glucose graph for insulin is shown in the following graph.

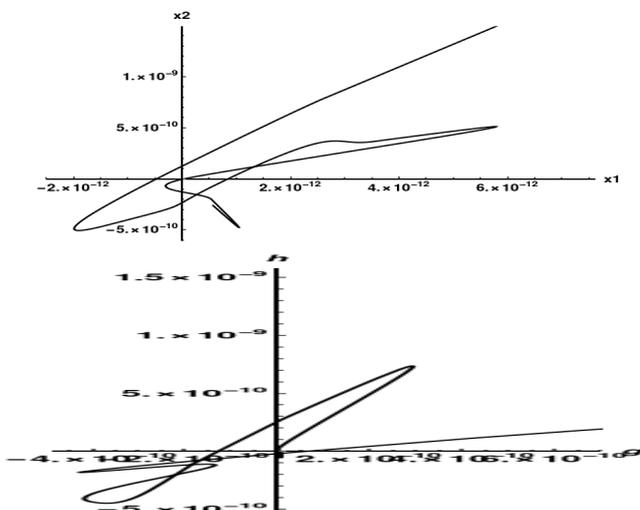


Fig. 3. Graph of x_2 with respect to x_1 .

Here the solution of the system represented in the graph of x_2 with respect to x_1 converges to the origin, this is in correspondence with the theoretically proven; indicating in this case that the total densities converge to the optimal values, which should be the objective of the home before nature.

III. CONCLUSIONS

1. The problem of conservation of the medium environment is of transcendental importance and a topical problem in scientific research, because of the current conditions, life on the planet is increasingly compromised, as shown in the first model presented.
2. Infrared values above acceptable values can cause chronic illness with unexpected effects. In which case $d > a$ and $bc > ad$, there will be no problems that affect our health.
3. Theorem 1 gives necessary and enough conditions for the density of the ozone layer and the intensity of the ultraviolet rays to converge to the optimal values making life on the planet comfortable.
4. Theorem 2 gives the procedure for reducing the periodic system in general in the wave-to-matrix system of the linear part to have constant coefficients and the coefficients of the non-linear part to be periodic.
5. The theorem 3 gives the necessary and enough conditions for the convergence of total concentrations to allowable concentrations.
6. Examples 1 and 2 show in practice the results obtained in theory, where in two the graphs show reality; showing in practice that the total densities of oxygen and carbon dioxide converge to the optimal values.

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